SUBJECT and GRADE	Mathematics Grade 11	
TERM 1	Week 4	
TOPIC	The Nature of Roots of Quadratic Equations	
AIMS OF LESSON	• Discuss the nature of roots of quadratic equations. • Solve word problems, using knowledge of equations.	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to *The Nature of Roots* chapter in your textbook.	Where you see this icon in the lesson you can click on it to see a video that would assist you in understanding the content that is being discussed

LESSON 1 a – Nature of Roots

INTRODUCTION

Nature of roots is a new concept in gr 11therefor we will tap in our pre-knowledge to guide us into understanding this new concept.

Pre-knowledge:

(1) Solving equations with the **FORMULA**

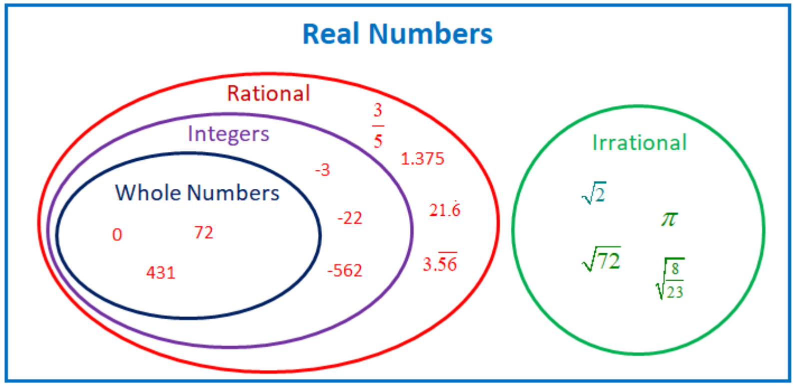
If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(2) The **roots** of an equation refer to the **solutions** of that equation.

The roots of a quadratic equation can be:

- ✓ **Real or non-real**
- ✓ **Rational or irrational**
- ✓ **Equal (two equal solutions) or unequal (two different solutions)**

(3) Understand that real numbers can be irrational or rational.



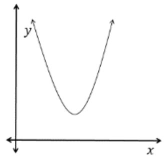
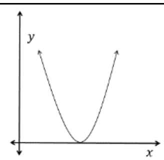
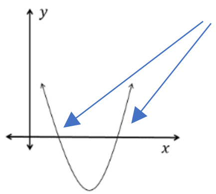
CONCEPTS AND SKILLS

Nature of Roots. **https://youtu.be/kQpgMOfmk-Q**

When we use the word **nature** in this context it refers to the features or character of numbers

The **roots** of an equation is the solutions of the equation, which also means the *x*-intercepts of a function can also be called roots

- When we discuss the nature of the roots, we are describing the types of numbers that the roots can be.
- The roots of any quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$ can be found by using the quadratic formula $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$
- The nature of roots can be found by determining the value of the discriminant also called delta (the part underneath the square root sign) $\Delta = \boldsymbol{b^2 - 4ac}$
- Three different possibilities of delta that can occur when we determine the nature of the roots, as summarized in the table below

$\Delta = \boldsymbol{b^2 - 4ac}$	**Nature of roots**	**Example**	**Graph**
$b^2 - 4ac < 0$	Non-real	$\sqrt{-17}$ or $\Delta = -17$	 No real roots
$b^2 - 4ac = 0$	Real. Equal, rational	$\sqrt{0}$ or $\Delta = 0$	 There will only be one repeated root. Also described as two equal roots The root will be real.
$b^2 - 4ac > 0$	Real, unequal, rational Real, unequal, irrational	$\sqrt{25}$ *or* $\Delta = 25$ perfect square $\sqrt{3}$ *or* $\Delta = 3$ not perfect square	 There will be two unequal real roots

There are three types of questions that we will need to solve when working with the nature of roots.

- Describe or name the nature of roots of a quadratic.
- Describe the nature of the roots when the roots are given.
- Determine the value of some unknown when the roots and the nature of the roots are given.

EXAMPLE 1

1.1

EXAMPLE 1.1 a

Solve for x if:

$\mathbf{2x^2 - 5x + 9 = 0}$

SOLUTION:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\boldsymbol{a = 2\,; b = -5\ ; c = 9}$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(9)}}{2(2)}$$

$$x = \frac{+5 \pm \sqrt{-47}}{4}$$

no real solution !

From this answer we can see that there are NO real roots.

EXAMPLE 1.1 b

Determine the nature of the roots if:

$\mathbf{2x^2 - 5x + 9 = 0}$

We can see that this nature of the roots ONLY depends on the value of Δ *in the formula.*

Therefore, we do not need to do the whole calculation if we are only interested in the nature of the roots.

We ONLY need to determine the value of Δ:

SOLUTION:

$\boldsymbol{a = 2\,; b = -5\ ; c = 9}$

$\Delta = b^2 - 4ac$
$= (-5)^2 - 4(2)(9)$
$= -47$

$\therefore$ Roots are:
Non-Real ($\Delta < 0$)

Remember that your expression must be in standard form so that ***a, b*** and ***c*** can be indicated and $\mathbf{b^2 - 4ac}$ can be determined

1.2

EXAMPLE 1.2a

Solve for x if:

$\boldsymbol{x^2 + 1 = 2x}$

$x^2 - 2x + 1 = 0$ [first write in standard form]

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\boldsymbol{a} = 1$ $\boldsymbol{b} = -2$ $\boldsymbol{c} = +1$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{0}}{2}$$

$x = 1$ or $x = 1$

From this answer we can see that there are real roots. There are TWO equal roots and that the roots are rational numbers.

EXAMPLE 1.2b

Determine the nature of the roots if:

$\boldsymbol{x^2 + 1 = 2x}$

We can see that this nature of the roots ONLY depends on the value of Δ *in the formula.*

Therefore, we do not need to do the whole calculation if we are only interested in the nature of the roots.

We can ONLY determine the value of Δ:

Solution

$x^2 - 2x + 1 = 0$ [first write in standard form]

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(1)(1)$

$= 0$

$\therefore$ Roots are:

- **Real** ($\Delta \geq \mathbf{0}$)
- **Rational** (Δ ***complete square***)
- **equal** ($\Delta = \mathbf{0}$)

Follow the steps when you need to determine the nature of the roots.

- Write the equation in standard form.
- Mindfully substitute *a, b* and *c*
- Calculate the discriminant, Δ
- Describe the nature of the roots

1.3

EXAMPLE 1. 3a

Solve for x if:

$x^2 + 3x - 10 = 0$

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1 \quad b = 3 \quad c = -10$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$x = 2 \qquad or \;\; x = -5$

From this answer we can see that there are real roots. There are TWO unequal roots and that the roots are rational numbers.

EXAMPLE 1.3 b

Determine the nature of the roots if:

$x^2 + 3x - 10 = 0$

We can see that this nature of the roots ONLY depends on the value of Δ in the formula.

Therefore, we do not need to do the whole calculation if we are only interested in the nature of the roots.

We can ONLY determine the value of Δ:

Solution

$\Delta = b^2 - 4ac$

$= (3)^2 - 4(1)(-10)$

$= 49$

∴ Roots are:

- **Real** ($\Delta \geq \mathbf{0}$)
- **rational** (Δ ***complete square***)
- **unequal** ($\Delta \neq \mathbf{0}$)

CAN YOU?

Determine the nature of the roots of the following equations.

1. $2x^2 + x - 3 = 0$
2. $x^2 + 4x = 2$
3. $x^2 + 3 = x$
4. $4x + \frac{1}{x} = 4$

Answers:

1. $\Delta = 25$ Roots are real, rational and unequal

2. $\Delta = 24$ Roots are real, irrational and unequal

3. $\Delta = -11$ Roots are non- real, imaginary there are no real roots

4. $\Delta = 0$ Roots are real, rational and equal

Example 2: Determine the nature of the roots of $f(x) = 0$, if $x = \frac{-m \pm \sqrt{(m-2)^2}}{2}$, and $m \neq \mathbf{2}$. **Solution:** $\Delta = (m-2)^2$ this is a perfect square ∴ Roots are: • **Real** $(\Delta > \mathbf{0})$ • **Rational** (Δ **is a perfect square)** • **Unequal** $(\Delta \neq \mathbf{0})$ **because** $\boldsymbol{m} \neq \mathbf{2}$
CAN YOU? Determine the nature of the roots of $f(x) = 0$, if $x = \frac{-k \pm \sqrt{-3k^2-4}}{2}$ **Answer:** $\Delta < \mathbf{0}$, **roots are non-real**
Example: 3 The roots of the equation $f(x) = 0$ is $x = \frac{4 \pm \sqrt{16-4m(-m+5)}}{2m}$ Determine the value(s) of m for which the roots are non-real. **Solution:** For non-real roots: $16 - 4m(-m+5) < 0$ $m^2 - 5m + 4 < 0$ $(m-4)(m-1) < 0$ $1 < m < 4$
CAN YOU? If $f(x) = 0$ have the roots, $x = \frac{-5 \pm \sqrt{3-12k^2}}{4}$, for which value(s) of k will the roots be equal: **Answer:** $k = \pm\frac{1}{2}$

LESSON 1b – **WORD PROBLEMS** **https://youtu.be/Mps38dhHXkc**

INTRODUCTION:

In general, learners in all grades battle with word problems

It is not possible to show you every possible word problem that you will ever encounter, but in this lesson we will discuss tools and skills that you can apply.

We will use mathematics to find solutions to real-life problems using our knowledge of equations.

CONCEPTS AND SKILLS

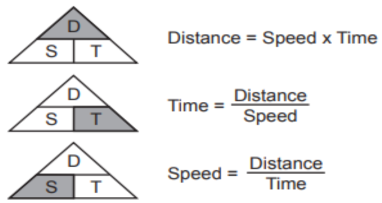
In this lesson we will use mathematical knowledge to find solutions to real-life problems using our content knowledge of equations.

General guidelines and important things to consider when solving word problems.

1. Read each problem three times. Firstly to determine what the problem is, then secondly to identify the "maths" words and thirdly to set up an equation.
2. Make sure you understand the story line. Use a mind map to summarise the problem.
3. At times drawing a sketch will make the problem clearer
4. Decide what exactly is being asked and make this the variable ready to be represented in the equation.
5. Make sure that you work in the same units for example km or m, hours or minutes etc.
6. If there are two unknowns, make the smaller one the variable chosen.
7. If two or more items are involved, express one item in terms of the other.
   For example: The boy is twice as old as his sister.
   The sister is younger, so she can be represented by the variable chosen (x being the most common) and the boy's age will then be 2x.
8. Set up your equation using any other information given in the statement.
9. Make sure the answer makes sense. For example, age cannot be negative.
10. Read the question again to check if you have answered the question before moving to the next problem.

We will now do some examples, read and study the guidelines as to ensure that you understand what skill or tools you can use in the following examples and exercise.

EXAMPLE 4

4.1	The product of two consecutive integers is 30 Determine the two integers. $x \text{ and } x+1$ $\therefore x(x+1) = 30$ Solve the equation: $\therefore x(x+1) = 30$ $x^2 + x - 30 = 0$ $(x+6)(x-5) = 0$ $x = -6 \text{ or } x = 5$ If $x = -6$ then $x+1$ will be equal to -5 If $x = 5$ then $x+1$ will be equal to 6 Therefor the answer to the question will be that the two numbers are either -6 and -5 OR 5 and 6	**Solution** Do you understand the terminology? (product, consecutive, integers) to make a story sum. Let the first number be x, the following/ consecutive number will then be $x+1$ Set up your equation using the information given in the statement. Simplify The two numbers must multiply together to make 30 $-6 \times -5 = 30$ and $5 \times 6 = 30$
4.2	Two runners set of at 06h00 in opposite directions. One runs at an average speed of 10km/h and the other runs at an average speed of 6km/h. At what time will they be 80km apart? $10x + 6x = 80$ $16x = 80$ $x = 5$	**Solution** You need to know the relationship between distance, speed and time. These triangles will help u to solve problems that involves distance, speed and time.  Let the time (number of hours) be x We can use the formula for distance if we have a value for speed and time.

It will take 5 hours for them to be 80km apart

$\therefore$ They will be 80 km apart at 11h00

Set up a table to help u solve this word problem

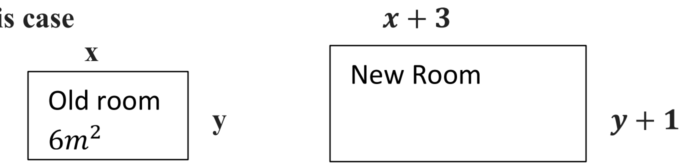
	Speed	distance	Time
Runner 1	10km / h	$10 \times x$	x
Runner 2	6km / h	$6 \times x$	x

4.3

A room is enlarged by increasing the length by 3m and the width by 1 m. The area of the new room is 3 times larger than that of the original room. Determine the original dimensions of the room if its area was $6m^2$

Solution

It is useful to graphically represent the word sum if it is possible like in this case



Let the dimensions of the **original room** be x and y $\therefore x \times y = 6m^2$

The new room would then be:

$x + 3$ *and* $y + 1$ (increased measurement)

$xy = 6$ and $(x+3)(y+1) = 18$

We can now work with simultaneous equations to solve for x and y

$xy = 6$

$\therefore y = \frac{6}{x}$

$$(x+3)(y+1) = 18$$

$$(x+3)\left(\frac{6}{x}+1\right) = 18$$

$$6 + x + \frac{18}{x} + 3 = 18$$

LCD = x

$$6x + x^2 + 18 + 3x = 18x$$

$$x^2 - 9x + 18 = 0$$

Write in standard form

$$(x-6)(x-3) = 0$$

$$x = 6 \text{ or } x = 3$$

$$\therefore y = 1 \text{ or } y = 2$$

The original dimensions were 6m and 1m

Or 3m and 2m

CAN YOU?

Solve the following word problems.: **Answers**

1.	The sum of two numbers is 20, while the product of the same numbers is 99. Find the two numbers	9 and 11
2.	Kim's mother is 6 times as old as Kim. The product of their ages is 150. What are their respective ages?	Kim is 5 and the mother 30 years old.
3.	Ricky has decided to treat her friends to coffee at the Corner Coffee House. Ricky paid R54,00 for four cups of cappuccino and three cups of filter coffee. If a cup of cappuccino costs R3,00 more than a cup of filter coffee, calculate how much a cup of each type of coffee costs?	A cup of cappuccino costs R9 and a cup of filter coffee costs R6.
4.	Two motorist travel to Port Elizabeth from Cape town in separate vehicles (They depart at the same time). The distance travelled is 720 km. The one motorist drives 10km per hour faster and arrives in Port Elizabeth one hour earlier than the other motorist. At what speed did the slower motorist travel.	80 km per hour
5	Two windmills work continuously and together can fill a reservoir in six days. Working separately, one windmill takes nine days longer than the other to fill the reservoir. Calculate how long it takes each windmill to fill the reservoir.	9 and 18 days respectively

Consolidation

- Having a solid understanding of the number system is very important in solving the nature of roots
- The roots of any quadratic equation $\boldsymbol{ax^2+bx+c=0}$ can be found by using the quadratic formula $\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$
- The nature of the roots of a quadratic equation depends on the value of the discriminant, $\Delta = b^2-4ac$
- The only way to become confident in solving word problems is to practice as many as possible on a regular basis.
- Solving problems is an integral part of life and an important skill to have.

<u>ACTIVITY</u>	**Mind Action Series**	**Via Africa**	**<u>Siyavula</u>**	Classroom Mathematics	Platinum
	Pg. 44 ex 10 Pg. 36 ex 8	Pg. 39 ex 11 Pg. 43 ex 14	End of chapter exercise Pg. 84 chapter 10 Pg. 64 Chapter 10	Pg. 65 ex 2.17, 2.18 Pg. 67 ex 2.18	Pg. 34 ex 7 Pg. 43 ex 13