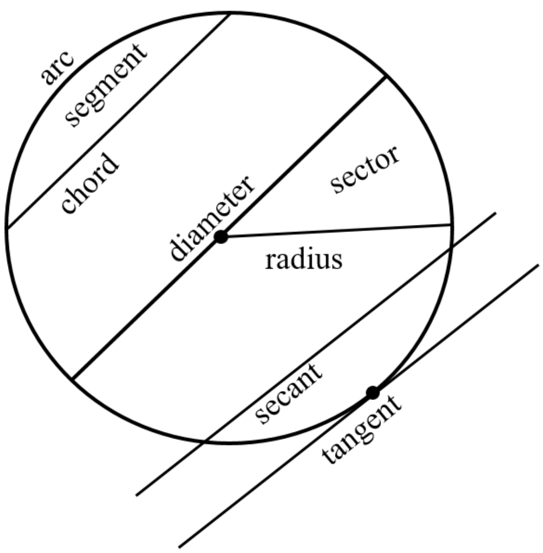
SUBJECT and GRADE	MATHEMATICS Gr 11
TERM 1	*Week 5*
TOPIC	**EUCLIDEAN GEOMETRY-LESSON 1**
AIMS OF LESSON	*State and prove the theorems for circle geometry.* • The line drawn perpendicular from the centre of a circle to a chord bisects the chord. • The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.
RESOURCES	
Paper based resources	***Digital resources***
Go to the chapter in your textbook on Circle Geometry.	*Chord in a circle* https://www.youtube.com/watch?v=J06Swxrvfkw *The perpendicular line from the centre of a circle bisects the chord and inverse* https://www.youtube.com/watch?v=gu_rGEf9Z2U https://www.youtube.com/watch?v=LOAe8vbxbp0 https://www.youtube.com/watch?v=XmkhPLFTh8Y *The perpendicular bisector line to the chord passes through the centre of the circle* https://www.youtube.com/watch?v=y5RPFTUj3xA *Example* https://www.youtube.com/watch?v=ma0qXCyxiQo

INTRODUCTION: *Introduction to Circle Geometry:*

- **Euclidean geometry**, the study of plane and solid figures on the basis of axioms and theorems employed by the Greek mathematician **Euclid** (300 BC).

BASIC CIRCLE TERMINOLOGY



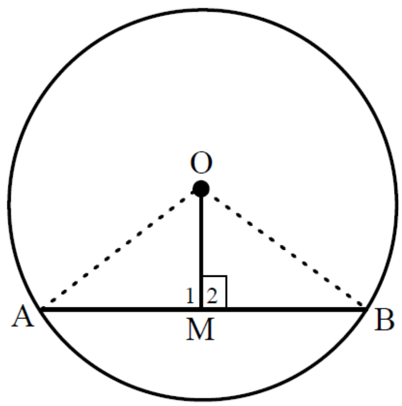
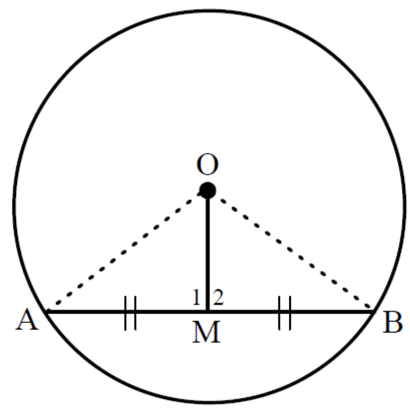
Radius:
A line from the center to any point on the Circumference of the circle.

Diameter:
A line passing through the center of the circle.
It is double the length of the radius.

Chord:
A line with end-points on the circumference.

Secant:
A line passing through two points on the circle.

Tangent:
A line touching the circle at only one point.

<table>
<tr><th colspan="3">CONCEPTS AND SKILLS</th></tr>
<tr>
<td colspan="2">THEOREM 1
The line drawn perpendicular from the centre of a circle to a chord bisects the chord.

If OM ⊥ AB then AM = MB</td>
<td>CONVERSE THEOREM 1
The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

If AM = MB then OM ⊥ AB</td>
</tr>
<tr>
<td>Acceptable REASON when you use the Theorem in the exam:</td>
<td>line from centre ⊥ to chord</td>
<td>Line from centre to midpoint of chord</td>
</tr>
<tr>
<td>PROOF OF THEOREMS</td>
<td>Given:
Circle with centre O with OM ⊥ AB.

What to prove: AM = MB

Construction: Join OA and OB

Proof:
In Δ OAM and Δ OBM:

(i) OA = OB radii
(ii) $\hat{M}_1 = \hat{M}_2 = 90°$ given
(iii) OM = OM common
∴ Δ OAM ≡ Δ OBM (RHS)

∴ AM = MB</td>
<td>Given:
Circle with centre O.
M is a point on chord AB such that AM = MB.

What to prove: OM ⊥ AB

Construction: Join OA and OB

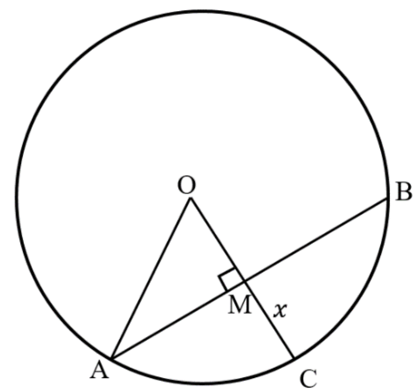
Proof:
In Δ OAM and Δ OBM:

(i) OA = OB radii
(ii) AM = BM given
(iii) OM = OM common
∴ Δ OAM ≡ Δ OBM (SSS)

∴ $\hat{M}_1 = \hat{M}_2 = 90°$ $\angle s$ on straight line</td>
</tr>
</table>

EXAMPLE 1

In the diagram is O is the centre of the circle, OM ⊥ AB and AB $= 8\ cm$. The radius of the circle is 5 cm.
Calculate the length of MC (x).



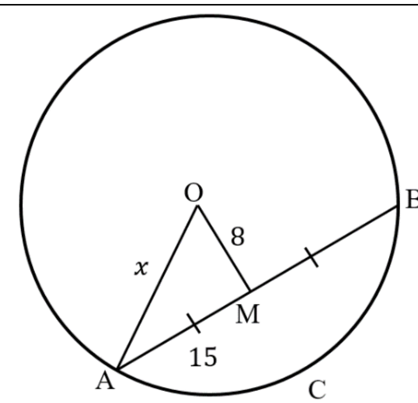
ANSWER:

Statement	***Reason***
AM=MB= 4	line from centre ⊥ to chord
$OM^2 = OA^2 - AM^2$ $OM^2 = (5)^2 - (4)^2$ $OM^2 = 9$ $OM = 3$	Pythagoras
$OC = OM + x$ $5 = 3 + x$ $\therefore x = 2$	OC= radius

EXAMPLE 2 - CAN YOU?

In the diagram is:

O is the centre of the circle, AM=MB= $15\ units$ and OM $= 8\ units$. Calculate the radius of the circle (x).



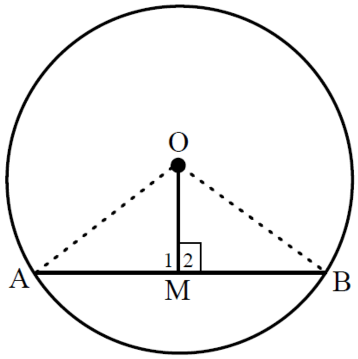
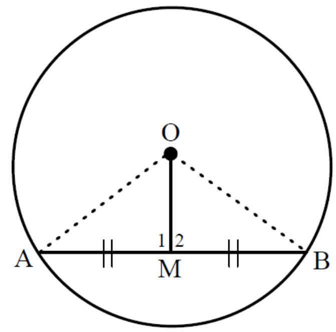
ANSWER:

Statement	***Reason***
$O\hat{M}A = 90°$	Line from centre to midpoint of chord
$OA^2 = OM^2 + AM^2$ $OA^2 = (8)^2 + (15)^2$ $OA^2 = 289$ $OA = 17 =$ radius	Pythagoras

ACTIVITIES/ ASSESSMENT	MIND ACTION SERIES (May 2012 Issue) Chapter 8 p 214 Excercise 1	CLASSROOM MATHEMATICS p 256 Exercise 10.1	VIA AFRICA Chapter 8 p 209 Exercise 1

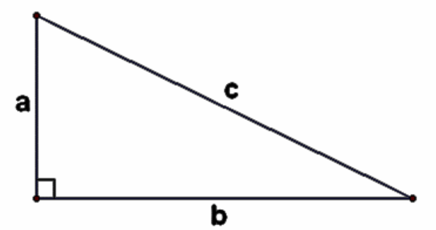
CONSOLIDATION

- *Know and understand the wording of the theorem(s).*
- *Learn the correct way of writing the reason for the Theorem(s)*

	
line from centre ⊥ to chord	***Line from centre to midpoint of chord***

- *Remember to use Pythagoras when you see these theorem(s).*

$$a^2 + b^2 = c^2$$



Geometry is creative rather than analytical, and students often have trouble making the leap between Algebra and Geometry. They are required to use their spatial and logical skills instead of the analytical skills they were accustomed to using in Algebra. With enough practice ***YOU CAN DO IT!***

VALUES

EUCLID

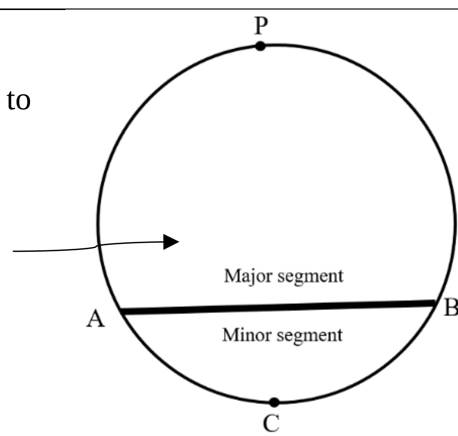
- Born 325 B.C.
- Greek Mathematician and father of Euclidean Geometry.
- Developed mathematical proof techniques that we know today.

SUBJECT and GRADE	**MATHEMATICS Gr 11**	
TERM 1	*Week 5*	
TOPIC	**EUCLIDEAN GEOMETRY-LESSON 2**	
AIMS OF LESSON	*State and prove the theorems for circle geometry.* In this lesson we will look at **TWO theorems** regarding the **ANGLE AT THE CENTRE** of the circle: • The angle at the centre is twice the angle at the circumference. • The angle in a semicircle is a right angle. As well as **TWO theorems** regarding angles on the circumference subtended by the **SAME OR EQUAL CHORDS**: • Angles in the same segment are equal. • Equal chords subtend equal angles at the circumference	
RESOURCES	***Paper based resources***	***Digital resources***
	Go to the chapter in your textbook on Circle Geometry.	• *Angle at the Centre is twice the angle at the circumference:* *https://www.youtube.com/watch?v=y7-yT5qUtN0* *https://www.youtube.com/watch?v=Y5VAApqtIZY* • *Angles in the same segment:* *https://www.youtube.com/watch?v=vPntRCGkZCo* *https://www.youtube.com/watch?v=BDqELk2xCPU* • *Angle in semi circle* *https://www.youtube.com/watch?v=oT7arIHd0D8* *https://www.youtube.com/watch?v=oT7arIHd0D8* • *General* *https://www.youtube.com/watch?v=BDqELk2xCPU* *https://www.youtube.com/watch?v=V711BEb06ck&t=12s*

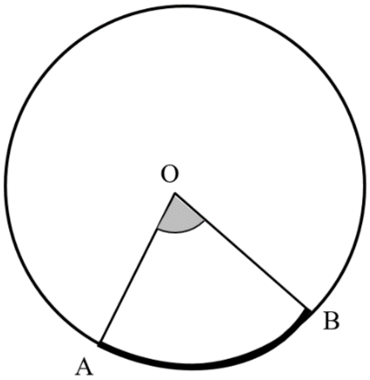
INTRODUCTION

BASIC CIRCLE TERMINOLOGY

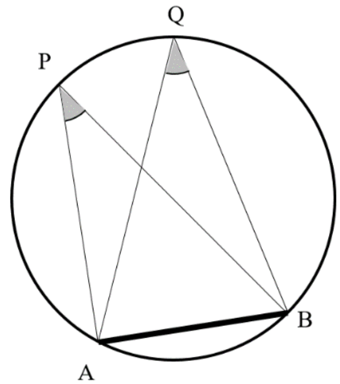
- **Semicircle:** half of a circle; the arc from one end of a diameter to the other.
- **Segment** of a circle can be defined as a region bounded by a chord and a corresponding arc lying between the chord's endpoints.



- **Subtended**: In geometry, an angle is subtended by an arc, line segment or any other section of a curve when its two rays pass through the endpoints of that arc, line segment or curve section.



$A\hat{O}B$ *is **subtended** by **arc** AB*



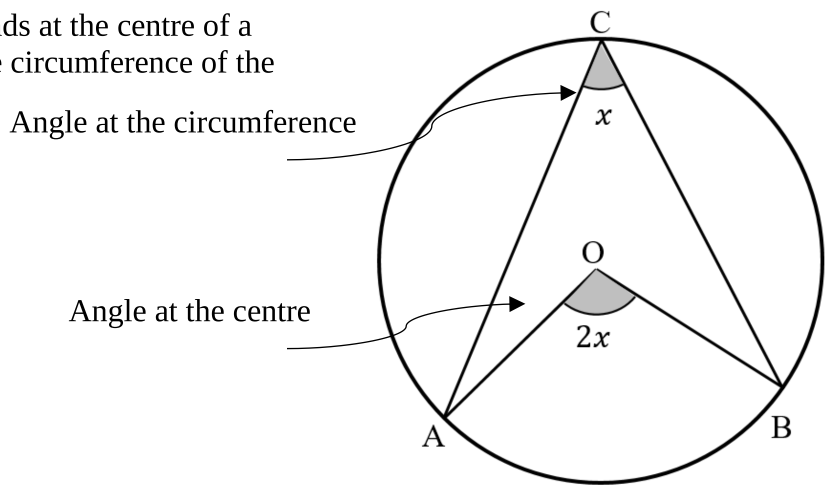
$\hat{P}$ *and* $\hat{Q}$ *is **subtended** by **chord** AB*

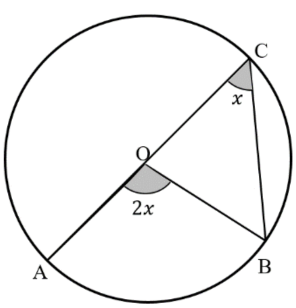
CONCEPTS AND SKILLS

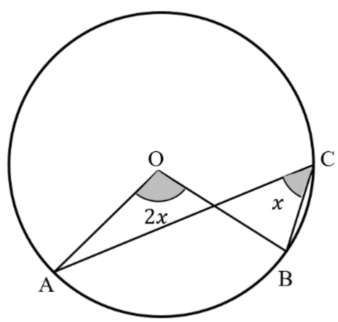
THEOREM 2

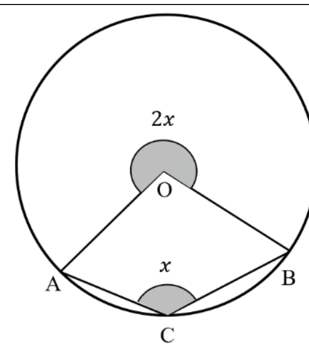
The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle

$A\hat{O}B = 2 \times \hat{C}$



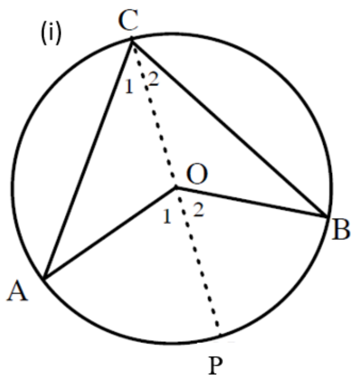


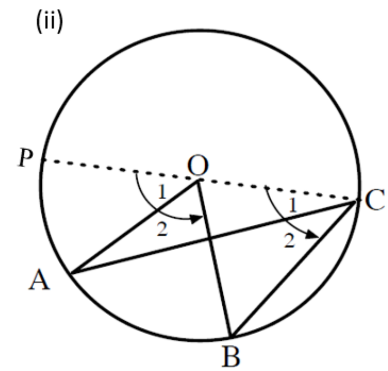




Acceptable REASON when you use the Theorem in the exam:	**$\angle$ at centre $= 2 \times \angle$ at circumference**

PROOF OF THEOREM





Given: Circle with centre O and A, B and C are all points on the circumference of the circle.

What to prove: $A\hat{O}B = 2 \times A\hat{C}B$

Construction: Join CO and produce to P.

Proof: (i)

Let $\widehat{C_1} = x$ and $\widehat{C_2} = y$

$\widehat{C_1} = \hat{A} = x$ ∠s opp. equal radii

$\widehat{O_1} = \widehat{C_1} + \hat{A} = 2x$ Ext ∠ of ΔOAC

Similarly, in Δ OCB:

$\widehat{O_2} = \widehat{C_2} + \hat{B} = 2y$

$\widehat{O_1} + \widehat{O_2} = 2x + 2y$

$= \mathbf{2(x + y)}$

$= \mathbf{2}\ (\widehat{C_1} + \widehat{C_2})$

$\therefore A\hat{O}B = 2 \times A\hat{C}B$

Proof: (ii)

Let $\widehat{C_1} = x$ and $\widehat{C_2} = y$

$\widehat{C_1} = \hat{A} = x$ ∠s opp. equal radii

$\widehat{O_1} = \widehat{C_1} + \hat{A} = 2x$ Ext ∠ of ΔOAC

Similarly, in Δ OCB:

$\widehat{O_2} = \widehat{C_2} + \hat{B} = 2y$

$\widehat{O_2} - \widehat{O_1} = 2y - 2x$

$= \mathbf{2(y - x)}$

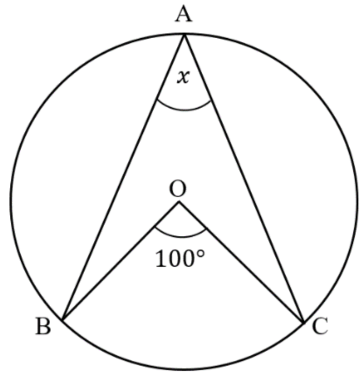
$= \mathbf{2}\ (\widehat{C_2} - \widehat{C_1})$

$\therefore A\hat{O}B = 2 \times A\hat{C}B$

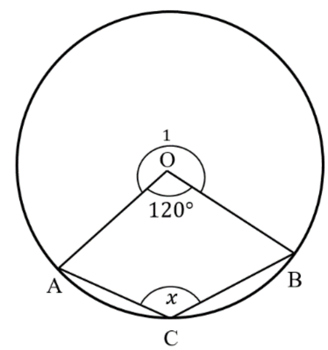
EXAMPLE 1

In the following diagrams, O is the centre of the circle. Determine, with reasons, the value of x.

1.1



1.2



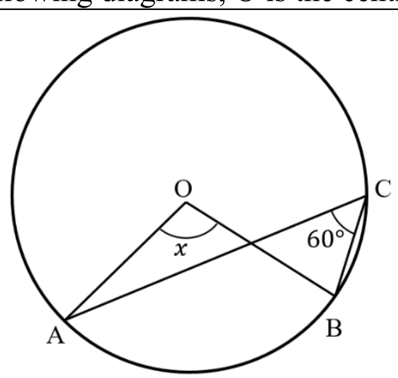
ANSWER:

Statement	***Reason***
1.1 $x = 50°$	$\angle$ at centre $= 2 \times \angle$ at circumference
1.2 $\widehat{O_1} = 240°$	revolution
$x = 120°$	$\angle$ at centre $= 2 \times \angle$ at circumference

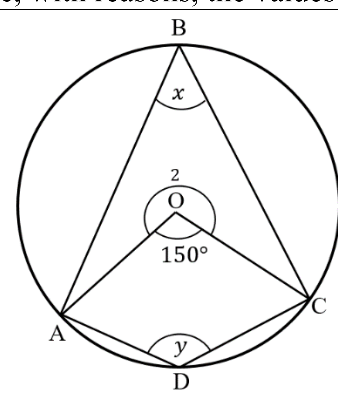
EXAMPLE 2 - CAN YOU?

In the following diagrams, O is the centre of the circle. Determine, with reasons, the values of x and y.

2.1



2.2



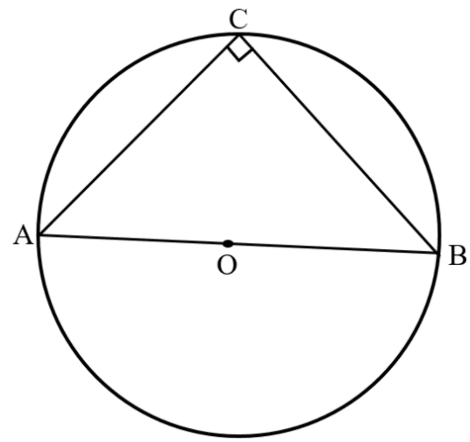
ANSWER:

Statement	***Reason***
2.1 $x = 120°$	$\angle$ at centre $= 2 \times \angle$ at circumference
2.2 $x = 75°$	$\angle$ at centre $= 2 \times \angle$ at circumference
$\widehat{O_2} = 210°$	revolution
$y = 105°$	$\angle$ at centre $= 2 \times \angle$ at circumference

THEOREM 3

The angle subtended at the circle by a diameter is a right angle.
(The angle in a semi-circle is 90°.)

You do not have to know the proof for this theorem for exam purposes, but I am sure you can think of one…



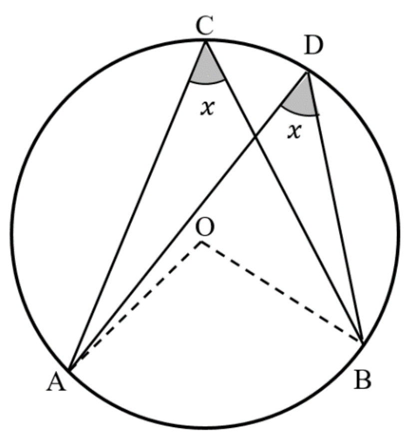
If AB is a diameter then $\hat{C} = 90°$

Acceptable REASON when you use the Theorem in the exam:	**$\angle$ in semi−circle**

THEOREM 4

An arc or chord of a circle subtends equal angles at the circumference of the circle. (angles in the same segment of the circle are equal if subtended by the same arc / chord)

You do not have to know the proof for this theorem for exam purposes, but I am sure you can think of one…



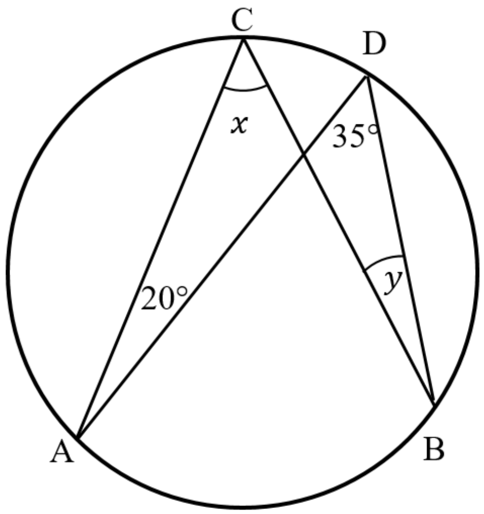
The angles on the circumference $\hat{C} = \hat{D}$ because both are subtended by arc AB.

Acceptable REASON when you use the Theorem in the exam:	**$\angle s$ in the same segment**

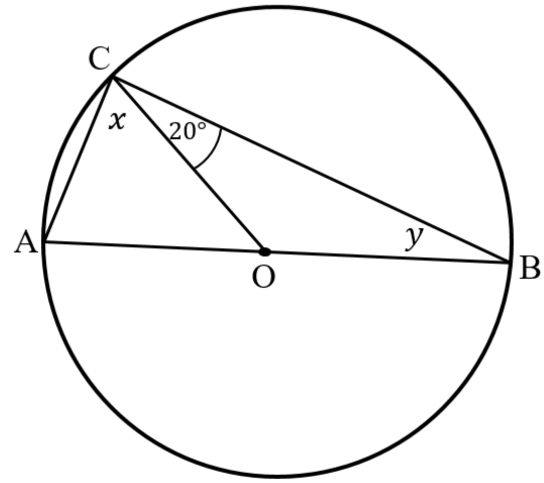
EXAMPLE 3

In the following diagrams, O is the centre of the circle. Determine, with reasons, the value of x and y.

3.1



3.2



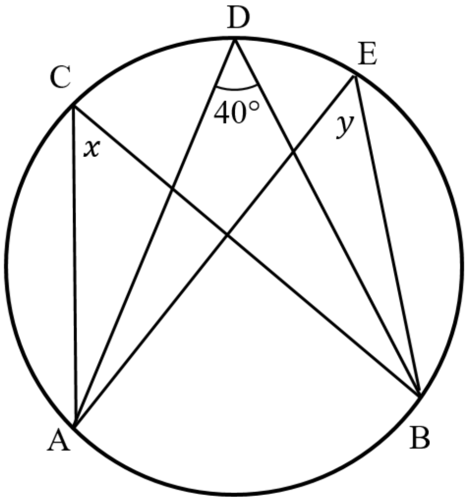
ANSWER:

Statement	*Reason*
3.1 $x = 35°$	∠s in the same segment
$y = 20°$	∠s in the same segment
3.2 $x = 70°$	∠ in semi−circle
$y = 20°$	∠ s opposite equal radii

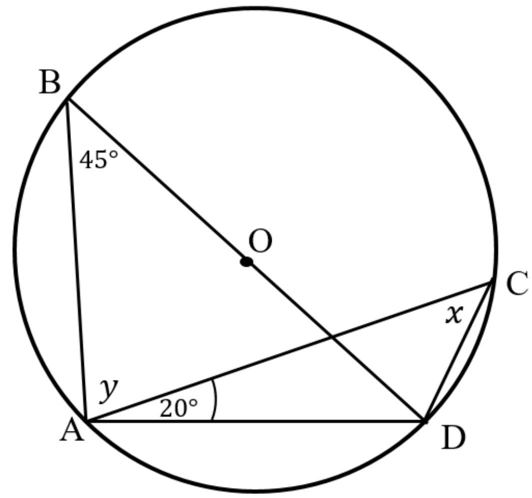
EXAMPLE 4 – CAN YOU?

In the following diagrams, O is the centre of the circle. Determine, with reasons, the value of x and y.

4.1



4.2



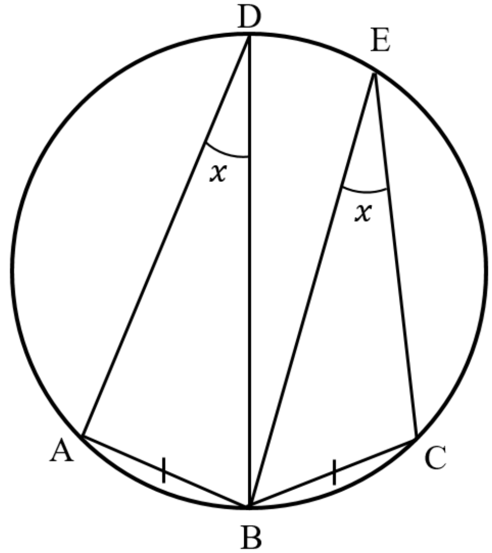
ANSWER:

Statement	*Reason*
4.1 $x = 40°$	$\angle$s in the same segment
$y = 40°$	$\angle$s in the same segment
4.2 $x = 45°$	$\angle$s in the same segment
$y = 70°$	$\angle$ in semi−circle

THEOREM 5

Equal chords subtend equal angles at the circumference.

You do not have to know the proof for this theorem for exam purposes.



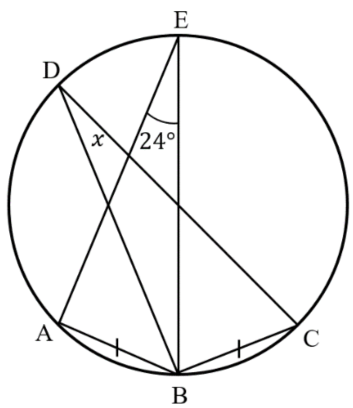
If AB=BC then $\hat{D} = \hat{E}$

Acceptable REASON when you use the Theorem in the exam:	**equal chords; equal ∠s**

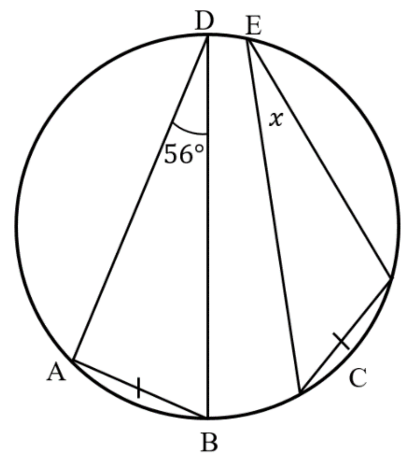
EXAMPLE 5

Determine, with reasons, the value of x.

5.1



5.2 – CAN YOU?



ANSWER:

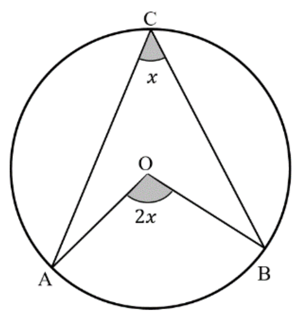
Statement	***Reason***
5.1 $x = 24°$	Equal chords ; equal ∠s
5.2 $x = 56°$	Equal chords ; equal ∠s

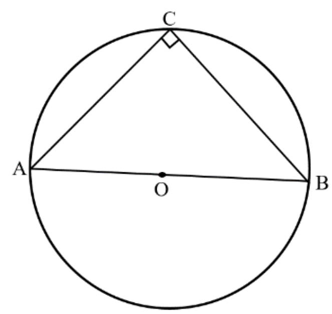
ACTIVITIES/ ASSESSMENT	MIND ACTION SERIES (May 2012 Issue) Chapter 8	CLASSROOM MATHEMATICS p 261	VIA AFRICA Chapter 8
	• *p* 217 *Exercise* 2 • *p 221 Exercise* 3 • *p223 Exercise 4* • *p 225 Exercise 5*	• Exercise 10.2	• *p* 211 Exercise 2 • *p* 214 Exercise 3

CONSOLIDATION

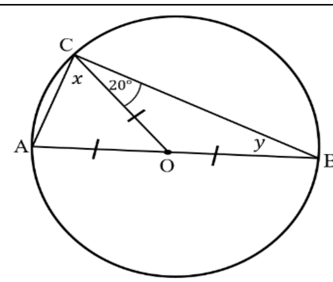
- *Know and understand the wording of the theorem(s).*
- *Learn the correct way of writing the reason for the Theorem(s)*

- *If the centre of the circle is given you must look for THESE theorems →*

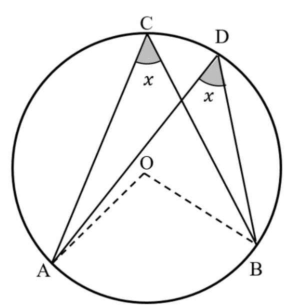




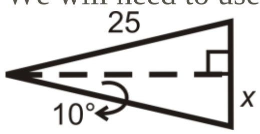
- *Also remember to mark all radii as this gives you isosceles triangles to work with.*

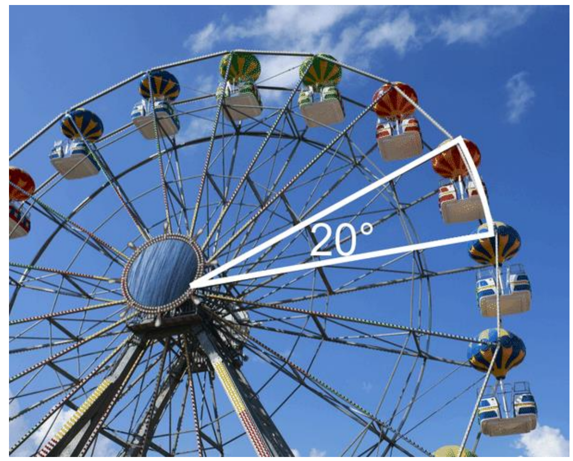


- *If you see there are angles on the circumference of the circle, remember to mark the angles subtended by the same arc!*



- *Geometry is creative rather than analytical, and students often have trouble making the leap between Algebra and Geometry. They are required to use their spatial and logical skills instead of the analytical skills they were accustomed to using in Algebra. With enough practice* ***YOU CAN DO IT****!*

<table>
<tr>
<td>VALUES</td>
<td>The Ferris wheel, radius 25, below had equally spaced seats, such that the central angle was 20°
Because the seats are 20∘ apart, there will be $\frac{360°}{20°}$ =18 seats.
It is important to have the seats evenly spaced for balance. To determine how far apart the adjacent seats are, use the triangle to the right.
We will need to use sine to find x and multiply by 2.




sin10°= $\frac{x}{25}$
$x = 4.3$

The total distance apart is 8.6 feet.

https://www.ck12.org/geometry/arcs-in-circles/lesson/Arcs-in-Circles-GEOM/</td>
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