SUBJECT and GRADE	Mathematics Grade 10
TERM 1	Week 1: Algebraic Expressions
TOPIC	Products and Factorisation
AIMS OF LESSON	To revise products and factorisation of grade 9 and to introduce gr 10 work on products and factorisation
RESOURCES	***Paper based resources*** *Please refer to the chapter in your textbook on Algebraic Expressions and Factorisation.*
INTRODUCTION	In this lesson we will revise the product of binomials which you have already started with in grade 9. This is a very important skill throughout algebra and a number of other sections of the curriculum. We will then go onto product of binomial and trinomial. This will be followed by revising factorization that was done in grade 9. **Binomial**: an expression with two terms. Remember terms are separated by a "+" or "-" sign, eg. $2a + 5b$ **Trinomial:** an expression with THREE terms, eg. $ax^2 + bx + c$
CONCEPTS/ SKILLS	**Products:** Changing a product expression (monomial) to a sum expression (polynomial) **Factorisation:** Changing a sum expression (polynomial) to a product expression (monomial)
Lesson 1	Products: **Revision of Gr 9 work**

Distributive law:

In an expression like $a(b+c)$, we can "spread" the a over the terms in the bracket:

$\boldsymbol{a(b+c) = a(b) + a(c) = ab + ac}$: *the distributive law*. Likewise, we have:

$\boldsymbol{(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd}$

1. Product of 2 Binomials: $\boldsymbol{(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd}$

Example 1: Find the following product: $(2a+5b)(3a-b)$

Solution: Method 1:

$= 2a(3a-b) + 5b(3a-b)$
$= 6a^2 - 2ab + 15ab - 5b^2$
$= 6a^2 + 13ab - 5b^2$

Each term in the one bracket must be multiplied by each term in second bracket: *the distributive law*

Method 2: $(2a+5b)(3a-b)$

Method 2 is often referred to as **FOIL**. It is an acronym for multiply:
First (each first term)
Outer (the outer 2 terms
Inner (the inner two terms)
Last (the last 2 terms).

First: $2a \times 3a$
Outer: $2a \times (-b)$
Inner: $5b \times 3a$
Last: $5b \times (-b)$

$\therefore (2a+5b)(3a-b) = 6a^2 - 2ab + 15ab - 5b^2$
$= 6a^2 + 13ab - 5b^2$

CAN YOU?

Exercise 1:

Find the following products:

1. $5a(a+b) - 3(a^2 - ab)$
2. $a^3(a^2 + 5a - 6)$
3. $\frac{1}{p}\left(p^2 - 2p + \frac{3}{p}\right)$
4. $(2a+5)(2a-1)$
5. $(2a-5)(2a-1)$
6. $(3x-5y)(2x+3y)$
7. $\left(5a + \frac{2}{5}b\right)\left(a - \frac{1}{5}b\right)$

Answers:

1. $2a^2 + 8ab$
2. $a^5 + 5a^4 - 6a^3$
3. $p - 2 + \frac{3}{p^2}$
4. $4a^2 + 8a - 5$
5. $4a^2 - 12a + 5$
6. $6x^2 - xy - 15y^2$
7. $5a^2 - \frac{3}{5}ab - \frac{2}{25}b^2$

2. Special case: Products leading to the difference of 2 squares: $(a+b)(a-b)=a^2-b^2$

Example 2: Find the following product: $(2x+3)(2x-3)$

Solution: If we apply FOIL: $(2x+3)(2x-3)$

$$=(2x)(2x)+2x(-3)+3(2x)+3(-3)$$

$$=4x^2-6x+6x-9$$
$$=4x^2-9$$

> **Inspection:** If the terms in the 2 brackets are the same with just the signs that differ, the answer will be: **(1st term)² – (2nd term)²**

Example 3: $\left(3a^2b+\frac{1}{2}xy^3\right)\left(3a^2b-\frac{1}{2}xy^3\right)$

$$=(3a^2b)^2-\left(\frac{1}{2}xy^3\right)^2$$

$$=9a^4b^2-\frac{1}{4}x^2y^6$$

3. Square of a binomial: $(a+b)^2=a^2+2ab+b^2$

Example 4: Find the following product:

$(x+3y)^2$

$$=(x+3y)(x+3y)$$

$$=x^2+3xy+3xy+9y^2$$

$$=x^2+6xy+9y^2$$

> **Inspection**: To square a binomial:
>
> **(1st term)² + [2 × (1st term) × (2nd term)] + (2nd term)²**

Example 5: Find the following product:

$(3ax-2y)^2$

$$=\mathbf{9a^2x^2-2(3ax.2y)+4y^2}=9a^2x^2-12axy+4y^2$$

Exercise 2:

Simplify the following expressions:*(use **Inspection** as far as possible)*

1. $(2a+5)(2a-5)$
2. $(x-6y^2)(x+6y^2)$
3. $\left(x^2a-\frac{1}{3}y^3b\right)\left(x^2a+\frac{1}{3}y^3b\right)$
4. $(x-y-z)(x-y+z)$
5. $(2a-5)^2$
6. $(2a+5)^2$
7. $(-x-3)^2$
8. $\left(x^{\frac{1}{2}}-9y\right)^2$
9. $[2(x+3y)]^2$
10. $2(3x+y)^2-3(x+2y)(x-2y)$

Answers:

1. $4a^2-25$
2. x^2-36y^4
3. $x^4a^2-\frac{1}{9}y^6b^2$
4. $x^2-2xy+y^2-z^2$
5. $4a^2-20a+25$
6. $4a^2+20a+25$
7. x^2+6x+9
8. $x-18x^{\frac{1}{2}}y+81y^2$
9. $4x^2 24xy+36y^2$
10. $15x^2+12xy+14y^2$

Lesson 2	Gr 10: **Product of binomial and trinomial**

4. Product of Binomial and Trinomial:

Trinomial: Is an expression with three terms, e.g. $2a + 5b - 3c$

Example 1: $(3x - 2y)(9x^2 - 6xy + 5y^2)$

Solution: $(3x - 2y)(9x^2 - 6xy + 5y^2)$

$= \mathbf{3x}(9x^2 - 6xy + 5y^2) - \mathbf{2y}(9x^2 - 6xy + 5y^2)$

Each term in the one bracket must be multiplied by each term in the second bracket: *the distributive law.*

$= 27x^3 - 18x^2y + 15xy^2 - 18x^2y + 12xy^2 - 10y^3$

$= 27x^3 - 36x^2y + 27xy^2 - 10y^3$

5. Products leading to sum/difference of 2 cubes:

Cube: number raised to the power 3, eg. x^3 (x cubed or x to the power 3)

Example 1: $(x + 2)(x^2 - 2x + 4)$

$= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8$

$= x^3 + 8$

Inspection: If we multiply a binomial and a trinomial with the following characteristics, then

$(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a}^2 - \boldsymbol{ab} + \boldsymbol{b}^2) = \boldsymbol{a}^3 + \boldsymbol{b}^3$: **sum of 2 cubes**

Example 2: $(2x + 3)(4x^2 - 6x + 9)$

$= (2x)^3 + 3^3$

$= 8x^3 + 27$

Inspection: If we multiply a binomial and a trinomial with the following characteristics, then

$(\boldsymbol{a} - \boldsymbol{b})(\boldsymbol{a}^2 + \boldsymbol{ab} + \boldsymbol{b}^2) = \boldsymbol{a}^3 - \boldsymbol{b}^3$: difference **of 2 cubes**

Example 3: $(2x - y)(4x^2 + 2xy + y^2)$

$= 8x^3 + 4x^2y + 2xy^2 - 4x^2y - 2xy^2 - y^3$

$= 8x^3 - y^3$

Example 4: $(2x - 3y)(4x^2 + 6xy + 9y^2)$

$= (2x)^3 - (3y)^3$

$= 8x^3 + 27y^3$

CAN YOU?

Exercise1:

Find the following products:

1. $(x - 1)(2x^2 + x - 1)$
2. $(x + 3y)(x^2 + 4xy + 9y^2)$
3. $(3s + 2t)(6s^2 - 4st + 3t^2)$
4. $(x + 3y)(x^2 - 3xy + 9y^2)$
5. $(2a - 4b)(4a^2 + 8ab + 16b^2)$
6. $(xy + 5)(x^2y^2 - 5xy + 25)$
7. $(9x^2 + 3xy + y^2)(3x - y)$
8. $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$
9. $(x^3 + 8)(x - 2)(x^2 + 2x + 4)$

Answers:

1. $2x^3 - x^2 - 2x + 1$
2. $x^3 + 7x^2y + 21xy^2 + 27y^3$
3. $18s^3 + st^2 + 6t^3$
4. $x^3 + 27y^3$
5. $8a^3 - 64b^3$
6. $x^3y^3 + 125$
7. $27x^3 - y^3$
8. $x^3 - \frac{1}{x^3}$
9. $x^6 - 64$

Lesson 3	**Mixed products** (Test your knowledge)

Revision Exercise: Simplify the following:

1. $-3x(x^2 - 3x + 2)$
2. $(2w - 1)(2w + 3)$
3. $(6m - 8p)(8p + 6m)$
4. $(3p - 1)^2$
5. $(x + 2)(x^2 - 3x + 2)$
6. $(3x + 2)(9x^2 - 6xy + 4y^2)$

Test:

Simplify the following products:

1. $2x(-x^2 + 3x - 2)$ (3)
2. $(3w + 7)(3w - 4)$ (3)
3. $(4p - q)^2$ (3)
4. $(5v - 3g)(3g + 5v)$ (2)
5. $(3x + 1)(x^2 - 3x - 1)$ (4)
6. $(2x - 3y)(4x^2 + 6xy + 9y^2)$ (2)
7. $(2p - q)(2p - 5q) - (q + 3p)(q - 3p)$ (4)
8. $(t - 4s)(t + 4s) + (3t - 4s)^2$ (4)

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Answers: Revision Exercise

1. $-3x^3 + 9x^2 - 6x$
2. $x^3 + 7x^2y + 21xy^2 + 27y^3$
3. $18s^3 + st^2 + 6t^3$
4. $x^3 + 27y^3$
5. $8a^3 - 64b^3$
6. $x^3y^3 + 125$
7. $27x^3 - y^3$
8. $x^3 - \frac{1}{x^3}$
9. $x^6 - 64$

Answers: Test

1. $-2x^3 - 6x^2 - 4x$
2. $9w^2 + 9w - 28$
3. $16p^2 - 8pq + q^2$
4. $25v^2 - 9g^2$
5. $3x^3 - 8x^2 - 6x - 1$
6. $8x^3 - 27y^3$
7. $13p^2 - 12pq + 4q^2$
8. $10t^2 - 24ts$

Lesson 4	Factorisation: **Revision of Gr 9 work**

Remember factorization is the reverse of finding the product – changing a sum expression (polynomial) to a product expression (monomial)

1. Common Factor (CF): $\boldsymbol{ab + ac = a(b + c)}$ OR $\boldsymbol{a(b + c) + d(b + c) = (b + c)(a + d)}$

> Take out the Common factor. Divide CF into each term to get other factor(s) - write this in a bracket

Factorise the following fully:

Example 1: $\mathbf{2}x + \mathbf{2}y$
$= \mathbf{2}(x + y)$

Example 2: $2\boldsymbol{x}^2 + 4\boldsymbol{x} - 3\boldsymbol{x}^3$
$= \boldsymbol{x}(2x + 4 - 3x^2)$

Example 3: $25a^4b^4 + 15ab^2 - 10a^2b^3$
$= 5ab^2(5a^3b^2 + 3 - 2ab)$

> HCF of 25, 15 and 10 is 5; CF of variables is **lowest power** of that variable if it is common to all terms

Example 4: $2x(x + 2y) - 3(x + 2y)$
$= (x + 2y)(2x - 3)$

> CF is an expression $(x + 2y)$. $(x + 2y)$ and $(2x - 3)$ are the **factors** of the expression.

Example 5: $3(2y - x) - 5x(x - 2y)$
$= 3(2y - x) + 5x(\boldsymbol{2y - x})$
$= (2y - x)(3 + 5x)$

> Change of sign to get CF $(2y - x)$

2. Difference of 2 squares (DOTS): $\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$

Example 6: $x^2 - b^2$
$= (x + b)(x - b)$

> DOTS: 2 brackets; $\left(\sqrt{term1} + \sqrt{term2}\right)\left(\sqrt{term1} - \sqrt{term2}\right)$

Example 7: $27y^3 - 3x^2y$
$= 3y(9y^2 - x^2)$
$= 3y(3y + x)(3y - x)$

> Always look for CF first

- There are **no** Real **factors** for the **Sum of 2 squares**, like $x^4 + 4$ is in its simplest form.
- To check whether your factors are correct, you can always do the multiplication as rough work.

CAN YOU?

Factorise fully:

1. $4ax + 2bx - 6cx$
2. $12xy^2 + 4xy - 3x^3y^2$
3. $t^2(4 - t) + 4(4 - t)$
4. $10b(x - y) + 5(x - y)$
5. $2a(3a + b) - 5b(3a + b)$
6. $x(a - 3) - 2(3 - a)$
7. $81x^2 - 36$
8. $x^4 - x^2$
9. $12a^3 - 75ax^2$
10. $-a^4 + 16$
11. $\frac{1}{4}b^2 - \frac{49}{16}$
12. $(a + x)^2 - y^2$
13. $4(2a - 3) + 36x^2(3 - 2a)$
14. $(x - 3)^2 - (x - 3)(x + 3)$
15. $(x - 1)(3x - 1) + (-9x^2 + 1)$

Answers:

1. $2x(2a + b - 3c$
2. $xy(12y + 4 - 3x^2y$
3. $(4 - t)(t^2 + 4)$
4. $5(x - y)(2b + 1)$
5. $(3a + b)(2a - 5b)$
6. $(a - 3)(x + 2)$
7. $9(3x - 2)(3x + 2)$
8. $x^2(x + 1)(x - 1)$
9. $3a(2a - 5x)(2a + 5x)$
10. $(4 + a^2)(2 + a)(2 - a)$
11. $\frac{1}{4}\left(b + \frac{7}{2}\right)\left(b - \frac{7}{2}\right)$
12. $(a + x + y)(a + x - y)$
13. $(2a - 3)(2 - 6x)(2 + 6x)$
14. $-6(x - 3)$
15. $-2(3x - 1)(x + 1)$

Lesson 5	Gr 10: Factorise: **4 terms by Grouping**

Here we work with 4 terms, as indicated.

Factorise completely:

Example 1: $xw + zy + zw + xy$

Solution: $= (xw + zw) + (zy + xy)$

$= w(x + z) + y(z + x)$

$= (x + z)(w + y)$

OR

$xw + zy + zw + xy$

Solution: $= (xw + xy) + (zy + zw)$

$= x(w + y) + z(y + w)$

$= (w + y)(x + z)$

- Group terms in pairs, so that each pair has a **common factor** and preferably with a + sign between the pairs
- Take out CF in each pair (considering sign changes) and continue as with CF as an expression

- Answers will be the same if each pair contains a CF
- NOTE: $(w + y)$ is the same as $(y + w)$

Example 2: $3ax - 3ay + x - y$

$= (3ax - 3ay) + (x - y)$

$= 3a(x - y) + (x - y)$

$= (x - y)(3a + \mathbf{1})$

NOTE: coefficient of $(x - y)$ is $+\mathbf{1}$

Example 3: $x^3 + 2x^2 - 4x - 8$

$= (x^3 + 2x^2) + (-4x - 8)$

$= x^2(x + 2) - 4(x + 2)$

$= (x + 2)(x^2 - 4)$

$= (x + 2)(x + 2)(x - 2)$

$= (x + 2)^2(x - 2)$

OR

$= (x^3 - 4x) + (2x^2 - 8)$

$= x(x^2 - 4) + 2(x^2 - 4)$

$= (x^2 - 4)(x + 2)$

$= (x + 2)(x - 2)(x + 2)$

$= (x + 2)^2(x - 2)$

$(x^2 - 4)$ can still be factorised further

CAN YOU?

Factorise completely**:**

1. $tx - ty + rx - ry$
2. $by - ax + bx - ay$
3. $3a^2 - ax + 6ab - 2bx$
4. $x^3 + 3x^2 - x - 3$
5. $2a - 1 - 2ab + b$
6. $6x^2 - ay - 2ax + 3xy$
7. $4a^2 - 4b^2 - b^3 + a^2b$
8. $6x^2 - x^5 - 6 + x^3$
9. $x^4 - x^2 - x^2y^2 + y^2$
10. $x^3 + y^3 + x + y$

Answers:
1. $(x - y)(t + r)$
2. $(b - a)(y + x)$
3. $(3a - x)(a + 2b)$
4. $(x + 3)(x - 1)(x + 1)$
5. $(2a - 1)(1 - b)$
6. $(3x - a)(2x + y)$
7. $(a - b)(a + b)(4 + b)$
8. $(x + 1)(x - 1)(6 - x^3)$
9. $(x + 1)(x - 1)(x + y)(x - y)$
10. $(x + y)(x^2 - xy + y^2)$

<table>
<tr><td>ACTIVITIES</td><td colspan="2">Consider other exercises from your Mathematics Textbook</td></tr>
<tr><td>CONSOLIDATION</td><td>PRODUCTS</td><td>FACTORISATION</td></tr>
<tr><td rowspan="2"></td><td colspan="2">Binomial: expression with 2 terms: $\boldsymbol{a + b}$ or $\boldsymbol{ax + c}$
Trinomial: expression with 3 terms: $\boldsymbol{a + b + c}$ or $\boldsymbol{ax^2 + bx + c}$</td></tr>
<tr><td>Distributive law: $\boldsymbol{a(b+c) = a(b) + a(c) = ab + ac}$

Product of 2 binomials:
$\boldsymbol{(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd}$

Products leading to the difference of 2 squares:
$\boldsymbol{(a+b)(a-b) = a^2 - b^2}$

Square of a binomial: $\boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$

Sum of 2 cubes:
$\boldsymbol{(a+b)(a^2 - ab + b^2) = a^3 + b^3}$

Difference of 2 cubes:
$\boldsymbol{(a-b)(a^2 + ab + b^2) = a^3 - b^3}$</td><td>Common Factor (CF):
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4 terms by Grouping:
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<tr><td>VALUES</td><td colspan="2">Dear learner. It is important to practice perseverance. Don't give up until you have arrived at the correct answer. Work daily at your Mathematics. Practice these skills to be able to do these fast in a test or examination.</td></tr>
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