SUBJECT and GRADE	MATHEMATICS GR 10
TERM 2	Week 1
TOPIC	ANALYTICAL GEOMETRY

AIMS OF LESSON

To calculate the:

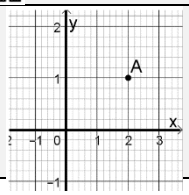
- Gradient of a line or coordinate of a point if the gradient is known.
- Distance between two known points or one of the coordinates of one of the points when the distance is known.

RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Analytical Geometry section in your Mathematics Textbook.	https://bit.ly/2K1kDtj https://www.siyavula.com

INTRODUCTION

To understand this new content area, you must understand the coordinates of point on the cartesian plane.

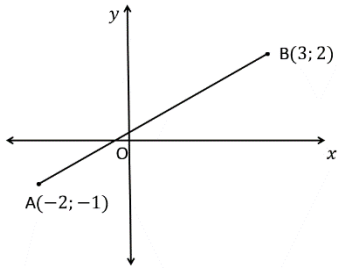
For example, the point $A(2;1)$ is plotted on the graph.



CONCEPTS AND SKILLS	• Gradient of line between two points • When lines are parallel, perpendicular and collinear
LESSON 1a Examples	**GRADIENT OF A LINE**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

1.1) Determine the gradient of AB



Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{2-(-1)}{3-(-2)}$$

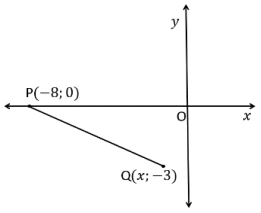
$$m_{AB} = \frac{2+1}{3+2}$$

$$m_{AB} = \frac{3}{5}$$

Steps:

1. Write down the formula for the gradient.
2. Label A as $(x_1;\ y_1)$ and B as $(x_2;\ y_2)$, then $x_1 = -2$; $y_1 = -1$; $x_2 = 3$ and $y_2 = 2$.
3. Substitute these values into the gradient formula.
4. Note that, $(-)(-) = +$, that is negative multiply a negative is a positive. That is why when you calculate $2 - (-1) = 2 + 1$.

1.2) Calculate the value of x if the gradient of PQ is $-\frac{1}{2}$



Solution:

$$m_{PQ} = \frac{-3-0}{x-(-8)}$$

$$\frac{-1}{2} = \frac{-3}{x+8}$$

$$-1(x+8) = 2 \times -3$$

$$-x - 8 = -6$$

$$-x = 2$$

$$x = -2$$

Steps:

1. Write down the formula for the gradient.
2. Label P as $(x_1;\ y_1)$ and Q as $(x_2;\ y_2)$, then $x_1 = -8$; $y_1 = 0$; $x_2 = x$ and $y_2 = -3$.
3. Also given that gradient of PQ is $-\frac{1}{2}$ this we can write as $m_{PQ} = -\frac{1}{2}$
4. Substitute these values into the gradient formula.
5. Cross multiply to get an equation withput fractions.
6. Solve for x.

EXERCISE: A

1. Find the gradient of AB if:

(a) $A(7;10)\ and\ B(-4;1)$
(b) $A(-5;-9)\ and\ B(3;2)$
(c) $A(x-3;y)\ and\ B(x;y+4)$

Answers:

□) $m_{AB} = 1$
b) $m_{AB} = \frac{11}{8}$
c) $m_{AB} = \frac{4}{3}$

2. Calculate the value of p, If the gradient of CD $= \frac{2}{3}$

(a) $C(16;2)and\ D(8;p)$
(b) $C(3;2p)and\ D(9;14)$

Answers:

2a) $\frac{-10}{3} = p$

2b) $5 = p$

LESSON 1b: **GRADIENT OF A LINE: Parallel and perpendicular lines**

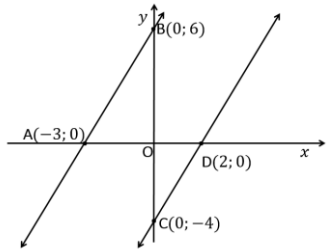
If two lines are parallel, their gradients will be equal. $AB \parallel CD \rightarrow \therefore m_{AB} = m_{CD}$

If two lines are perpendicular, the product of their gradients will be equal to -1

$AB \perp CD \rightarrow \therefore m_{AB} \times m_{CD} = -1$

Examples:

1.3) Prove that $AB \parallel CD$.



Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{6 - 0}{0 - (-3)}$$

$$m_{AB} = \frac{6}{3}$$

$$m_{AB} = 2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{CD} = \frac{0 - (-4)}{2 - 0}$$

$$m_{CD} = \frac{4}{2}$$

$$m_{CD} = 2$$

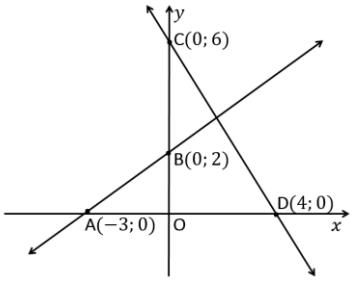
$$m_{AB} = m_{CD}$$

$$\therefore AB \parallel CD$$

Steps:

1. Write down the formula for the gradient.
2. Label the points with $A(x_1;\ y_1)$ and B as $(x_2;\ y_2)$ and work out the gradient for AB.
3. Label the points with $C(x_1;\ y_1)$ and D as $(x_2;\ y_2)$ and work out the gradient for CD.
4. Compare the two answers. If they are the same, the lines will be parallel.

1.4) Prove that $AB \perp CD$.



Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{2 - 0}{0 - (-3)}$$

$$m_{AB} = \frac{2}{3}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{CD} = \frac{6 - 0}{0 - 4}$$

$$m_{CD} = \frac{6}{-4}$$

$$m_{CD} = -\frac{3}{2}$$

$$m_{AB} \times m_{CD} = \frac{2}{3} \times -\frac{3}{2} = -1$$

$$\therefore AB \perp CD$$

Steps:

1. Write down the formula for the gradient.
2. Label the points with $A(x_1;\ y_1)$ and B as $(x_2;\ y_2)$ and work out the gradient for AB.
3. Label the points with $D(x_1;\ y_1)$ and C as $(x_2;\ y_2)$ and work out the gradient for CD.
4. Compare the two answers. If the product of the gradients are equal to -1, the two lines will be perpendicular.

EXERCISE B

1. Determine whether *AB* and *CD* are parallel, perpendicular or neither

(a) $A(3; -4), B(5; 2), C(-1; -1), D(7; 23)$

(b) $A(3; -4), B(5; 2), C(-1; -1), D(0; -4)$

(c) $A(3; -4), B(5; 2), C(1; 1), D(-2; 2)$

Exercise B Answers:

1a) $\therefore AB \parallel CD$

1b) AB is not parallel to CD

and AB is not perpendicular to CD

1c) AB is not parallel to CD

but $m_{AB} \times m_{CD} = -1$

$\therefore AB \perp CD$

LESSON 1c: **COLLINEAR POINTS**

If three points are collinear (in a straight line) the gradients between the points will be the same.

If $m_{AB} = m_{BC} = m_{AC}$ → A , B and C are in a straight line

Examples:

1.5)
Determine if $E(0;3)$, $F(-2;5)$ and $G(2;1)$ are collinear or not.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{EF} = \frac{5-3}{-2-0}$$

$$m_{EF} = \frac{2}{-2}$$

$$= -1$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{FG} = \frac{5-1}{-2-2}$$

$$m_{FG} = \frac{4}{-4}$$

$$= -1$$

$m_{EF} = m_{FG}$

$\therefore E, F \text{ and } G$ are co−linear (in a straight line)

Steps:

1. Write down the formula for the gradient.
2. Label the points with $E(x_1;\ y_1)$ and F as $(x_2;\ y_2)$ and work out the gradient for EF.
3. Label the points with $G(x_1;\ y_1)$ and F as $(x_2;\ y_2)$ and work out the gradient for FG.
4. Compare the two answers. If they are the same, the points are in a straight line (collinear).

1.6)
Determine c if $A(1;3)$, $B(2;2)$ and $C(-1;c)$ are collinear (in a straight line).

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{2-3}{2-1}$$

$$= \frac{-1}{1}$$

$$m_{BC} = \frac{2-c}{2-(-1)}$$

$$= \frac{2-c}{3}$$

$$m_{AB} = m_{BC}$$

$$\frac{-1}{1} = \frac{2-c}{3}$$

$$1(2-c) = -1 \times 3$$

$$2 - c = -3$$

$$-c = -5$$

$$c = 5$$

Steps:

1. Label the points with $A(x_1;\ y_1)$ and B as $(x_2;\ y_2)$ and work out the gradient for AB.
2. Label the points with $C(x_1;\ y_1)$ and B as $(x_2;\ y_2)$ and work out the gradient for BC.
3. The gradients are the same, therefore we can write it down as an equation.
4. Solve for c, by cross multiplication.

EXERCISE C

1. Determine whether the following points lie on the same straight line (collinear):

(a) $E(0;3), F(-2;5),\ G(2;1)$
(b) $H(-3;-5),\ I(0;0),\ J(6;10)$
(c) $K(-6;2),\ L(-3;1),\ M(1;-1)$

2.Points P$(-6;2)$, $Q(2;-2)$ and $R(-3;r)$ lie on a straight line. Find the value of r.

3.Line PQ with P$(-1;-7)$ and $Q(q;0)$ has a gradient of 1. Find q.

Exercise C Answers:

1a) $E, F \text{ and } G$ lies on the same straight line

1b) $H, I \text{ and } J$ lies on the same straight line

1c) $K, L \text{ and } M$ does NOT lie on the same straight line, **no**t collinear

2. $r = \frac{1}{2}$

3. $q = 6$

ACTIVITIES/ ASSESSMENT

Mind Action Series	Platinum	Classroom Mathematics	Everything Mathematics
Ex: 3; Pg 220	***Ex: 3; Pg 193*** ***Ex: 4; Pg 195***	***Ex: 11.4; Pg257***	***Ex: 8.3; Pg 296*** ***Ex: 8.4; Pg 304***