SUBJECT and GRADE	MATHEMATICS GR 10
TERM 2	Week 3
TOPIC	FUNCTIONS – THE PARABOLA $y = ax^2 + q$
AIMS OF LESSON	To:

- Introduce you to the shape, the standard form of the equation of a parabola, and the impact of "a" and "q".
- Sketch the parabola using the characteristics of the parabola.
- Determine the Domain and Range for the parabola.
- Determine the equation of the parabola when the sketch is given.

RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Chapter on Functions and then to the section on the parabola in your Mathematics Textbook.	https://bit.ly/3eAQEpW ; https://bit.ly/3btNnH2 https://bit.ly/3eFGWm7 ; https://bit.ly/2VXz6Mr

INTRODUCTION:

Dear learner in this section on functions and graphs you will be learning about three different functions. The Parabola, hyperbola and exponential function. In this lesson we will be focusing on the parabola.

In grade 9 you have encountered the straight line. The standard form of the straight line is: $y = mx + c$

On a particular sketch you could be given $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$. Instead of giving the function in this way, we can give each of these straight lines a name, so let's say the first one is f and the second one is g. We can write these functions in what is called functional notation as, $f(x) = 2x + 3$ and $g(x) = -\frac{1}{2}x + 3$.

Example 1:

Consider the function $f(x) = x^2$,

x	-3	-2	-1	0	1	2	3
$f(x)$							

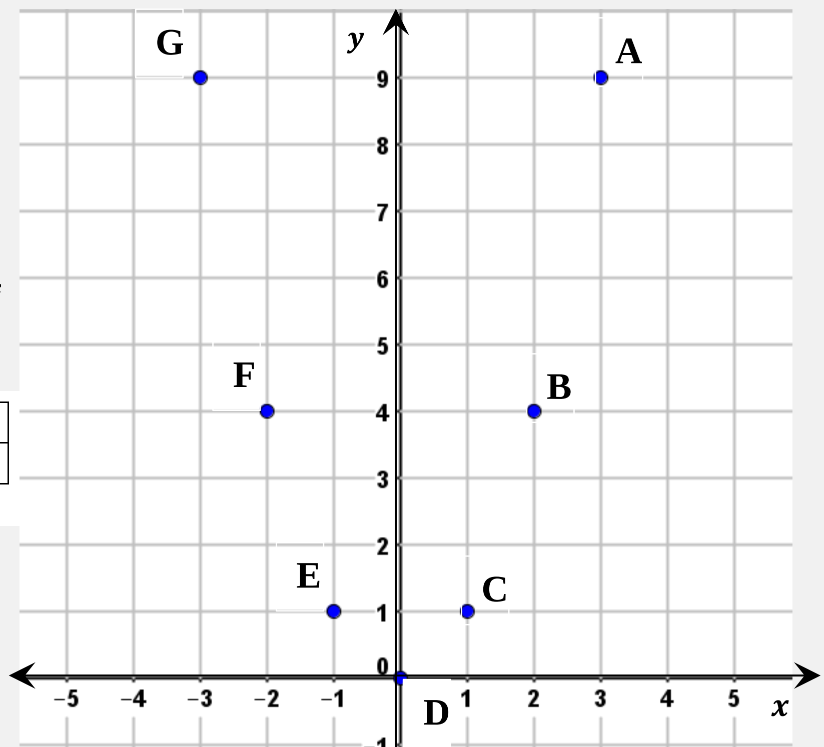
a) Complete the above table.

b) The points have been plotted on the given Cartesian plane, try on your own, and check if they are correct.

Solution:

a)

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

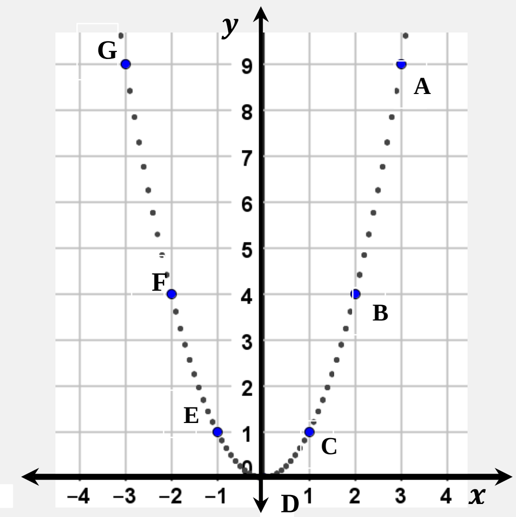


Note:
$f(x)$ **is the** y**-coordinate of each point. The points are named so that it becomes easy to refer to the points. So the following is obtained from the table:**
A(3; 9) , B(2; 4), C(3; 9) , D(0; 0), E(-1; 1), F(-2; 4) and G(-3; 9)

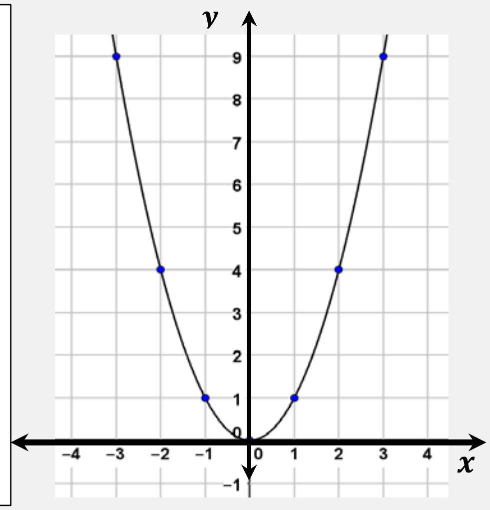
How to plot A(3; 9) on the Cartesian plane???.

Goto 3 on the x- axis, then move 9 units up along the y-axis.

All the other points can be plotted in the same way



If you were to take more x-values and calculate the corresponding y-values for the function, $f(x) = x^2$ and then plot the points you will find the image in the sketch on the left-handside. appearing.
I am sure you will agree if more and more points were determined and plotted the graph on the right-handside will eventually emerge.



DEFINITION OF A PARABOLA	A graph of a quadratic function i.e. $y = ax^2 + q$ where q is a real number and $a \neq 0$ is called a **parabola**

We need to investigate the impact of "a" and "q" on the graph of the parabola. That is, given a graph of a parabola how does changing the value of "a"and "q", change the original graph of the parabola.

Investigate Impact of "a":

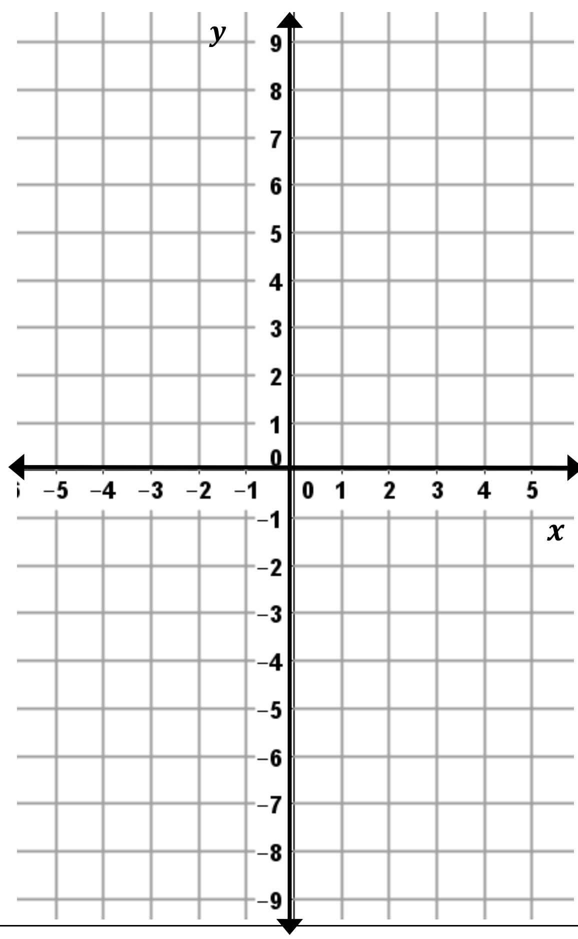
Consider the functions:

$f(x) = x^2$, $g(x) = 2x^2$, $h(x) = \frac{1}{2}x^2$ and $k(x) = -x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$							
$g(x)$							
$h(x)$							
$k(x)$							

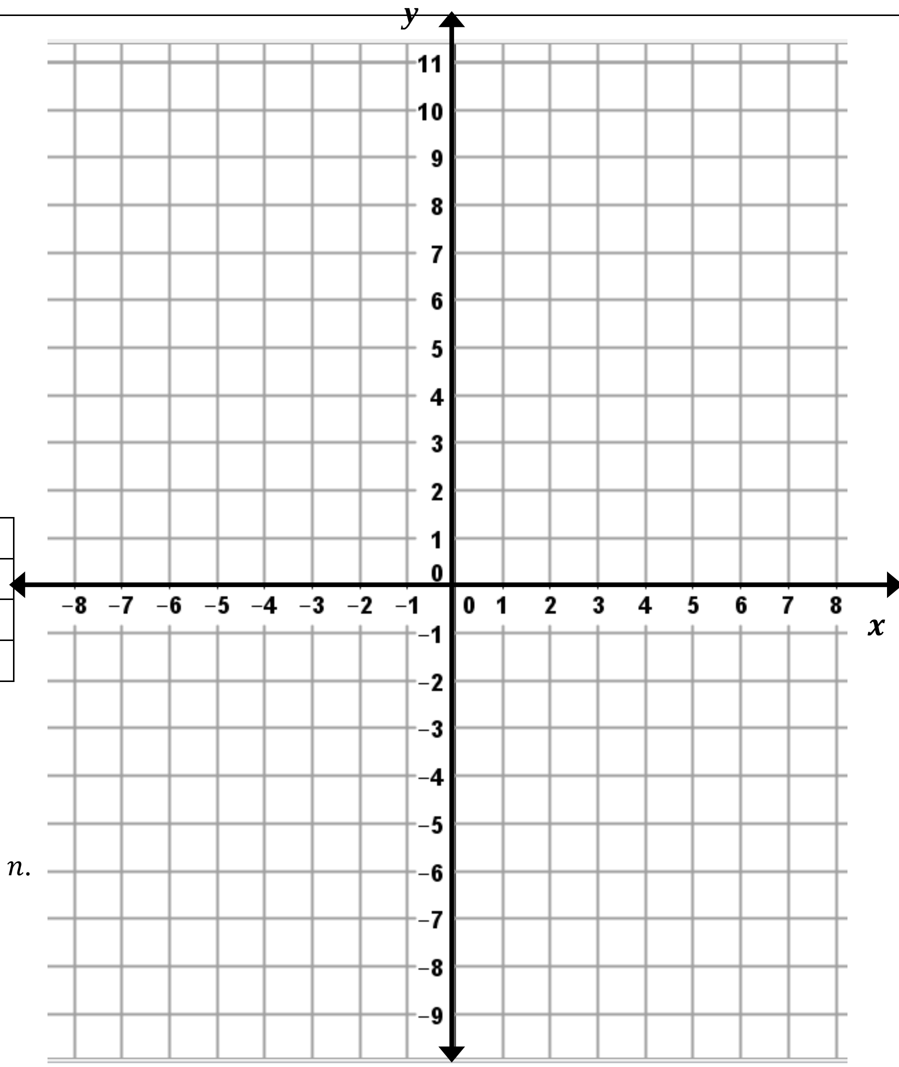
a) Complete the table above.
b) Plot the points of $f(x)$ on the Cartesian plane.
c) Join the points plotted in b).
d) Repeat (b) & (c) for functions g, h and k
e) What are you able to conclude of the impact of "a" on the graph of the parabola?

Compare the graphs of f , g, h and k and how they have changed and how the value of "a" for these graphs have changed.



Investigate Impact of "q":

Consider the functions:
$f(x) = x^2$,
$m(x) = x^2 + 2$, and
$n(x) = x^2 - 2$.



a) Complete the table below.

x	-3	-2	-1	0	1	2	3
$f(x)$							
$m(x)$							
$n(x)$							

b) Plot the points on the Cartesian plane for f.
c) Join the points plotted in b)
d) .Repeat (b) & (c) for functions m and n.
e) What are you able to conclude of the impact of "q" On the graph of the parabola.

Conclusion: In the equation $\boldsymbol{y} = ax^2 + \boldsymbol{q}$

If $\boldsymbol{a > 0}$, i.e. if a is positive The graph has a '**smiley face** 	If $\boldsymbol{a < 0}$, i.e. if a is negative the graph has a '**sad face'**. 	If $\boldsymbol{q > 0}$, i.e. if $q$is positive Then q is the number of units the function, $f(x) = x^2$ is shifted **up**.	If $\boldsymbol{q < 0}$, i.e. if q is negative Then q is the number of units the function $f(x) = x^2$ is shifted **down.**

Terminology:

Turning Point: The turning point of the parabola is the point where the parabola turns or change direction. This point has an x-coordinate and a y-coordinate.

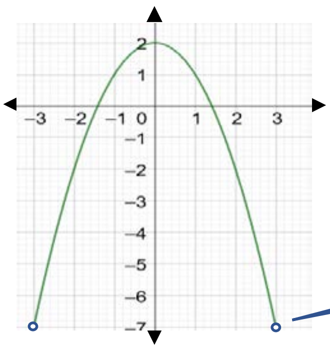
Axes of Symmetry: The axes of symmetry is the vertical line, that divides the parabola so that the one half is a mirror image of the other. The axes of symmetry is the x-coordinate of the turning point. In Grade 10 you will be working with the parabola of standard form $\boldsymbol{y} = ax^2 + \boldsymbol{q}$, this Parabola is symmetrical about the $\boldsymbol{y}$ -axis. The equation of the axes of symmetry of this form of the parabola, is $x = 0$.

Minimum/Maximum Value:
The minimum value (if $\boldsymbol{a > 0}$) or maximum value ($\boldsymbol{if\ a < 0}$) is the $\boldsymbol{y}$ -coordinate of the Turning Point.
***x* - intercept:** This is where the graph intersects the x-axes.
***y* - intercept:** This is where the graph intersects the y-axes.

Domain is the set of x-values for which the graph or function has been defined.
Range is the set of y-values for which the graph or function has been defined.

Domain and Range of a Function

Example 2: Given



Write down: a) Domain
b) Range

a) Domain: $-3 < x < 3$ or an interval $x \in (-3; 3)$

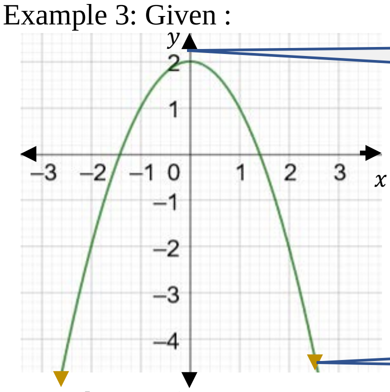
Domain are all x-values between -3 and 3, not including -3 and 3.

As this parabola has open circles at both ends, the graph stops at these points with -3 and 3 not included as points on the graph.

b) Range: $-7 < y \leq 2$ or $y \in (-7; 2]$

-7 is not included in the range of this function.because -3 and 3 is not in the domain of the function

Example 3: Given :



Write down: a) Range
b) Domain

a) Range: $y \leq 2$ or $y \in (-\infty; 2]$

Note that the maximum value of this function is 2. Thus all real x-values will have a corresponding y-value smaller than 2. As this graph continues indefinitely the range is from 2 to $-\infty$.

b) Domain: $-\infty < x < \infty$ or $x \in R$

This arrow indicate that the graph is continuing beyond this page. This function is defined for all real x-values.

Sketch the Parabola:

You are expected to sketch the parabola by using the characteristics of a parabola, and not by using point by point plotting. When sketching a parabola ensure that enough information is given on the sketch so that if you had the sketch you are able to determine the equation of the parabola, if that was not given.

To sketch the parabola:

1. Establish the shape of the graph by identifying if the equation is of the form, $y = ax^2 + q$, if it is, you will know that it is a parabola. The sign of "a", will indicate that the parabola is either concave up or down.
2. Determine the x- intercepts and y-intercepts. Some parabolas do not have an x-intercept. If a parabola does not have an x- intercept, determine the coordinates of any other point on the graph and plot this on the sketch.
3. Determine the coordinates of the Turning point. The turning point for, $y = ax^2 + q$ is (0; q). This point is also the y- intercept of this parabola.

EXAMPLE 4

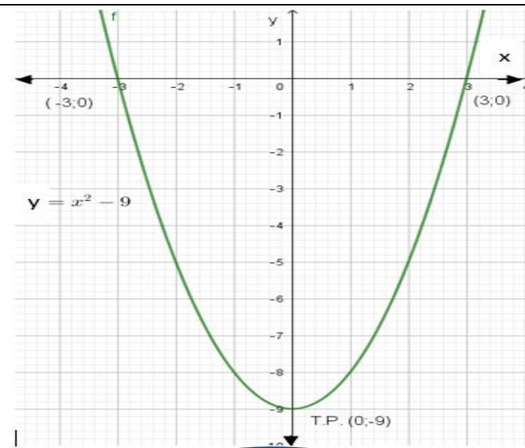
1. Sketch the graph of $y = x^2 - 9$.

2. Write down the :
2.1 domain
2.2 range

3.1 Does the function have a maximum or minimum value?

3.2 Write down the equation of the axes of symmetry for the graph?

Solution:
1.
Shape of graph is:
because $\boldsymbol{a > 0}$
& form of equation
Is $y = ax^2 + q$



***x*-intercept**: let $y = 0$
$0 = x^2 - 9$
$0 = (x - 3)(x + 3)$
$(x - 3) = 0 \;\; or \;\; (x + 3) = 0$
$x = 3 \quad or \quad x = -3$
$(-3; 0) \;\; and \;\; (3; 0)$
***y*-intercept**: let $x = 0$
$y = (0)^2 - 9$
$y = -9 \;\; \therefore \;\; (0; -9)$
Turning Point: (0; -9)

When given a parabola of form $\boldsymbol{y = ax^2 + q}$.

The **y-intercept** & **Turning point** is the same point.

This point is: (0; q)

2.1 Domain: $x \in R$
2.2 $-9 \le y < \infty$ or $y \in [-9; \infty)$

3.1 Minimum Value of -9
3.2 $x = 0$

EXERCISE A : (Answer on last page, check after you have done it)

1. Draw the graph of $y = -x^2 + 16$

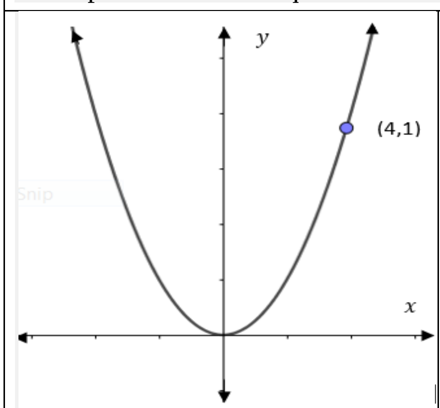
2. Write down the :
2.1 domain
2.2 range

3.1 Does the function have a maximum or minimum value?
3.2 Write down the equation for the axes of symmetry for the graph?
3.3 Write down the coordinates of the turning point?

Determining the equation of a parabola:

To find the equation of a parabola you need to find the values of a and q for the equation $y = ax^2 + q$.

Example 5 Find the equation of the parabola below.

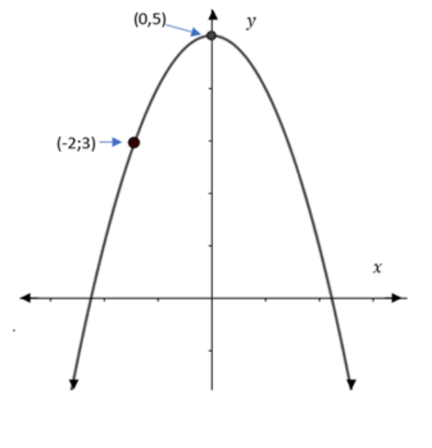


Method of Solving
1. Identify the y-intercept/turning point, the y-coordinate will give you the value of q.
2. To find $\boldsymbol{a}$ we substitute the coordinates of a point on the graph into the equation.

Solution:
y-intercept (0; 0) $\therefore \quad q = 0$
$\therefore \quad y = ax^2 + 0$

Substitute the point $(4; 1)$ into $y = ax^2$
$\therefore \quad 1 = a(4)^2$
$\therefore \quad 1 = 16a$
$(\div 16) \qquad \therefore \quad a = \frac{1}{16}$
Substitute $a = \frac{1}{16}$ and $q = 0$
into the standard form of the equation
$\therefore$ The equation is $y = \frac{1}{16}x^2$

Example 6 Determine the equation of the parabola passing through (0; 5) and (-2; 3).



Method of Solving

1. Identify the y-intercept, the y-coordinate/turning point will give you the value of q.
2. To find $\boldsymbol{a}$ we substitute the coordinates of a point on the graph into the equation.

Solution:

y – intercept is (0; 5) $\therefore$ $q = 5$

Substitute $q = 5$ into $y = ax^2 + q$

$\therefore$ $y = ax^2 + 5$

Substitute the point $(-2; 3)$ into $y = ax^2 + 5$

$\therefore$ $3 = a(-2)^2 + 5$

$\therefore$ $3 = 4a + 5$

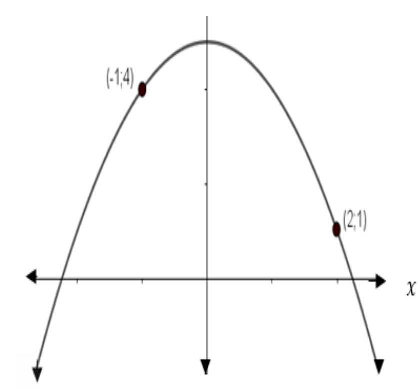
$\therefore$ $4a = -2$

$\therefore$ $a = \frac{-1}{2}$

Substitute $a = \frac{-1}{2}$ and $q = 5$ into the standard form, the

The equation is: $y = \frac{-1}{2}x^2 + 5$

Example 7 Find the equation of the parabola which passes through the points (-1; 4) and (2; 1).



Method of Solving

We cannot find $\boldsymbol{a}$ or $\boldsymbol{q}$ directly.
To find $\boldsymbol{a}$ and $\boldsymbol{q}$ we substitute each of the 2 points (-1 ; 4) and (2 ; 1) into the equation $y = ax^2 + q$.
You will have two equations each with two unknowns.
Now solve the two linear equations
Simultaneously for $\boldsymbol{a}$ and $\boldsymbol{q}$.

Substitute (-1; 4) into $4 = a(-1)^2 + q$

Substitute (2; 1) into $1 = a(2)^2 + q$

we need two equations to find $\boldsymbol{a}$ and $\boldsymbol{q}$.

$4 = 1a + q$ …………………(1)

$1 = 4a + q$ …………………..(2)

Subtract equation numbered (2) from equation numbered (1)

$\therefore$ $3 = -3a$

$\therefore$ $a = -1$

Substitute $(a = -1)$ into either equation (1) or (2)

$4 = (-1)(-1)^2 + q$

$4 = -1(+1) + q$

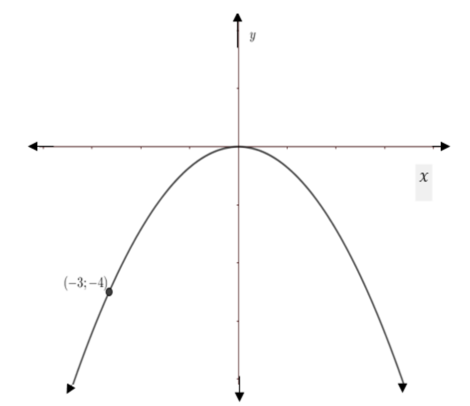
$4 = -1 + q$

$\therefore$ $q = 5$

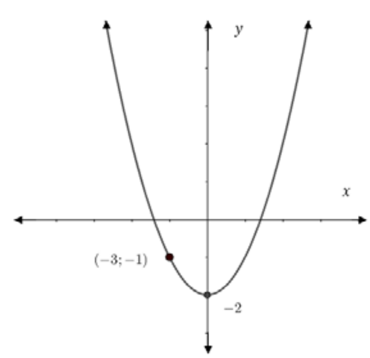
$\therefore$ The equation is $y = -x^2 + 5$

EXERCISE B Determine the equations of the following parabolas below.

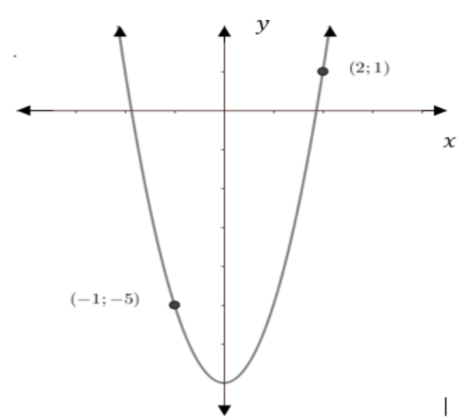
1. The graph passing through (-3; -4)

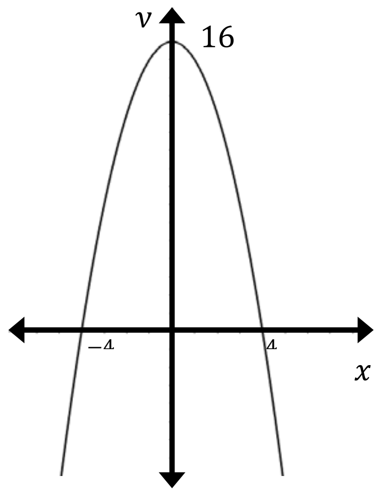


2. Through the point -2 and (-3; -1)



3 Passing through the points (2;1) and (-1; -5)



Exercise A: Answers: 1.  	2.1 Domain: $x \in R$ 2.2 $-\infty < y \leq 16$ or $y \in (-\infty;\ 16]$ 3.1 Maximum Value of 16 3.2 $x = 0$ 3.3 Turning Point (0; 16)	**EXERCISE B** Answers: 1. $y = -\frac{4}{9}x^2$ 2. $y = \frac{1}{9}x^2 - 2$ 3. $y = 2x^2 - 7$

Consolidation:

1. If $a > 0$ the parabola is concave up. If $a < 0$, then the parabola is concave down. If "a", is positive for two different parabolas, then the parabola with the bigger "a" , value is more stretched or steep.
2. The value of, "q", gives the number of units the parabola is moved above or below the x axes.
3. To sketch $y = ax^2 + q$, determine
   a) the shape of the graph
   b) x-intercepts if it has and
   c) Turning point which is (0; q)
4. To find the equation of a parabola, $y = ax^2 + q$,
   4.1 you must be given two bits of information to determine "a" and "q".
   4.2 Turning point and y-intercept is the same point, (0; q)
   4.3 Domain: $x \in \Re$ and
   4.4 Range is either $y \in (-\infty;\ q]$ if $a < 0$ or $y \in [q; \infty)$ if $a < 0$

EXERCISES on the Parabola

Siyavula
pg's.185-188 Chapter 5 No.2;6;10(b);
Combination problems: pg.189 -190 No.'s 13

Mind Action Series Grade 10
pg 116 Ex 2 a - g

Classroom Mathematics pg's.142-178:
pg. 159 Ex. 8.1 No.1 & 2
pg.162 Ex. 8.2 No.1; pg. 170 Ex. 8.3 No.1;
pg. 172 Ex. 8.4 No.1(b);2(b),3(b); pg. 178 Ex. 8.5 No.1;3

Platinum Math: pg.133 Ex. 6 &7; Combination Exercise pg.141 Ex 11

Mind Action Series
Pg.146 Ex.9 No.1(b); pg. 149 Ex.10 No. 2; 3(c) &(f); pg.150 (a); (d); & (f)
Combination Exercise pg.161 -162 (a); (c); (e); (g)