SUBJECT and GRADE	MATHEMATICS GR 10
TERM 2	Week 6
TOPIC	TRIGONOMETRY
AIMS OF LESSONS	

To:
- Revise the theorem of Pythagoras
- Define the trig ratios in terms of a right-angled triangle.
- Define the reciprocals of the trig ratios in a right-angled triangle.
- Determine Ratios for special angles.

RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Trigonometry section in your Mathematics Textbook.	*https://youtu.be/pfiy4diHlok* *https://youtu.be/KPU7ugbYKp0*

INTRODUCTION

Trigonometry is a section of Mathematics where we will concentrate on the inter-relationship between the lengths of the sides and the sizes of the angles in a triangle. In Gr 10 we will focus on a right-angled triangle.

CONCEPTS AND SKILLS

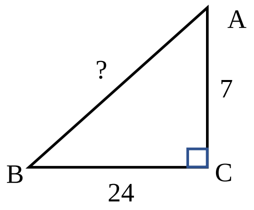
LESSON 1**: REVISE THEOREM OF PYTHAGORAS**

Theorem of Pythagoas: **In ΔABC with ∠C = 90° :** $c^2 = a^2 + b^2$

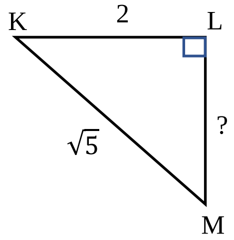
Example 1

Determine the length of the missing side:

a)



b)



Solutions:

a)

$AB^2 = AC^2 + BC^2$

$= (7)^2 + (24)^2$

$= 49 + 576$

$= 625$

$\therefore AB = 25$

b)

$KM^2 = KL^2 + LM^2$

$(\sqrt{5})^2 = (2)^2 + LM^2$

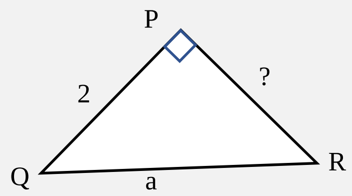
$5 = 4 + LM^2$

$LM^2 = 1$

$\therefore LM = 1$

Steps:
1. Apply Thm of Pyth on ΔABC
2. Substitute values and simplify
3. Determine square root to get answer.

Can you, determine the length of PR?



Solution: $\sqrt{a^2 - 4}$

Exercise 1

Determine the lengths of the missing sides:

A) In Δ ABC is ∠C = 90°, AB = 13 and BC = 5. Determine AC

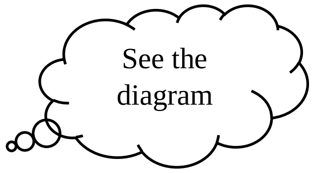
B) In Δ JKL is ∠K = 90°, JK = p and KL = 1. Determine JL

Solutions: A) 12 B) $\sqrt{p^2 + 1}$

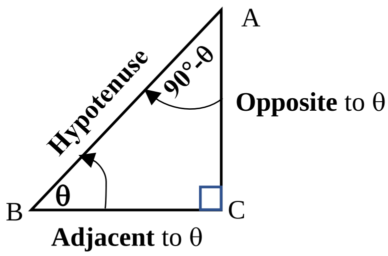
LESSON 2: DEFINE TRIG RATIOS IN TERMS OF A RIGHT-ANGLED TRIANGLE

The following must be noted for Trigonometry:

- We use Greek letters (θ, α, β, etc.) to denote the angles in trigonometry.
- θ and (90°- θ) are a pair of complementary angles in a right-angled triangle.
- The side across the 90° angle (the longest side) is called the HYPOTENUSE (h)
- The side touching angle θ is ADJACENT (a) to θ.
- The side across angle θ is OPPOSITE (o) to θ.
- There are SIX (6) relationships of the sides in a right-angled triangle with respect to one-another.



Naming sides in a right-angled triangle:



SUMMARY:

sin θ $= \frac{o}{h}$, **cos θ** $= \frac{a}{h}$, **tan θ** $= \frac{o}{a}$

Soh Cah Toa

OR

sinoh cosah tanoa

(make your OWN rhyme to remember the ratios as they are **VERY IMPORTANT**)

Defining the trig ratios in a right-angled triangle:

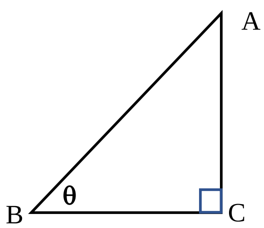
1. The ratio $\frac{\textbf{opposite to }\theta}{\textbf{hypotenuse}}$ is called the **sine** of angle θ and we write it as **sin θ**
2. The ratio $\frac{\textbf{adjacent to }\theta}{\textbf{hypotenuse}}$ is called the **cosine** of angle θ and we write it as **cos θ**
3. The ratio $\frac{\textbf{opposite to }\theta}{\textbf{adjacent to }\theta}$ is called the **tangent** of angle θ and we write it as **tan θ**

For our purposes in Gr 10 – 12 we will mainly use these THREE ratios. The next 3 are called the **RECIPROCALS**

4. The ratio $\frac{\textbf{adjacent to }\theta}{\textbf{opposite to }\theta}$ is called the **cotangent** of angle θ and we write it as **cot θ** $= \frac{1}{\tan\theta}$
5. The ratio $\frac{\textbf{hypotenuse}}{\textbf{adjacent to }\theta}$ is called the **secant** of angle θ and we write it as **sec θ** $= \frac{1}{\cos\theta}$
6. The ratio $\frac{\textbf{hypotenuse}}{\textbf{opposite to }\theta}$ is called the **cosecant** of angle θ and we write it as **cosec** θ $= \frac{1}{\sin\theta}$

Example: 2

A) Write the trig ratios for sin θ, cos θ and tan θ using the following triangle:



Solutions:

A) $\sin\theta = \frac{o}{h} = \frac{AC}{AB}$

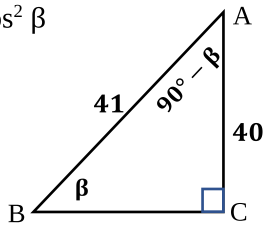
$\cos\theta = \frac{a}{h} = \frac{BC}{AB}$

$\tan\theta = \frac{o}{a} = \frac{AC}{BC}$

B) Use ΔABC to determine the value of :

1. BC
2. tan β
3. cos β
4. sin (90° - β)
5. $\cos^2 \beta$

Note: We write $(\cos \beta)^2$ as $\cos^2 \beta$



B) 1. BC = 9 …..Pyth

2. $\tan \beta = \frac{o}{a} = \frac{AC}{BC} = \frac{40}{9} = 4\frac{4}{9}$

3. $\cos \beta = \frac{a}{h} = \frac{BC}{AB} = \frac{9}{41}$

4. $\sin(90° - \beta) = \frac{BC}{AB} = \frac{9}{41}$

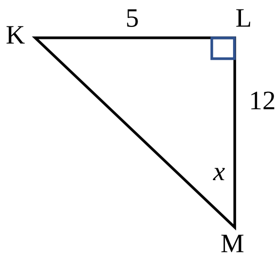
5. $\cos^2 \beta = \left(\frac{a}{h}\right)^2 = \left(\frac{BC}{AB}\right)^2 = \left(\frac{9}{41}\right)^2 = \frac{81}{1681}$

Note: cos β = sin(90° - β) !!!! because they are co-ratios/ functions

Exercise 2
CAN YOU study the previous example and complete;

Use ΔKLM to determine the value of:

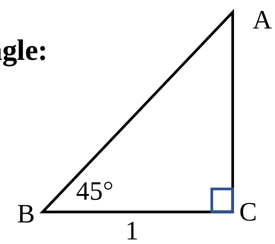
1. KM
2. $\cos^2 x$
3. $\tan x$
4. $\sin x + \cos x$



Solutions:
1. 13 2. $\frac{144}{169}$ 3. $\frac{5}{12}$ 4. $1\frac{4}{13}$

LESSON 3: **RATIOS OF SPECIAL ANGLES**

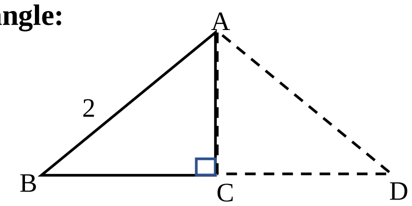
45° triangle:



In ΔABC is ∠C = 90°, ∠B = 45° and BC = 1 unit.

Give, with reasons the:
1. size of ∠A
2. length of AC
3. length of AB

60°/ 30° triangle:



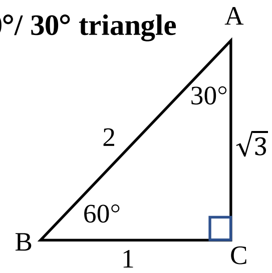
ΔABD is equilateral with side AB = 2 units.
Draw AC ⊥ BD

Why is:
1. ∠B = 60°?
2. BC = 1 unit?

Determine the:
3. size of ∠BAC
4. length of AC

SUMMARY:

60°/ 30° triangle



$\sin 60° = \frac{o}{h} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{a}{h} = \frac{1}{2}$

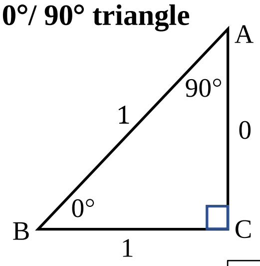
$\tan 60° = \frac{o}{a} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\sin 30° = \frac{o}{h} = \frac{1}{2}$

$\cos 30° = \frac{a}{h} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{o}{a} = \frac{1}{\sqrt{3}}$

0°/ 90° triangle



$\sin 0° = \frac{o}{h} = \frac{0}{1} = 0$

$\cos 0° = \frac{a}{h} = \frac{1}{1} = 1$

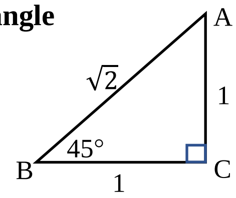
$\tan 0° = \frac{o}{a} = \frac{0}{1} = 0$

$\sin 90° = \frac{o}{h} = \frac{1}{1} = 1$

$\cos 90° = \frac{a}{h} = \frac{0}{1} = 0$

$\tan 90° = \frac{o}{a} = \frac{1}{0} =$ **undefined**

45° triangle



$\sin 45° = \frac{o}{h} = \frac{1}{\sqrt{2}}$

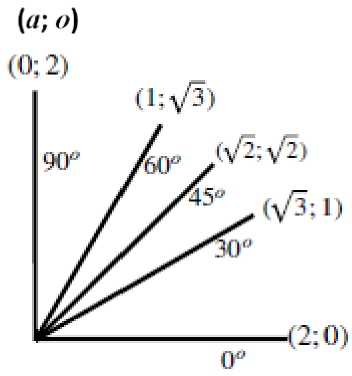
$\cos 45° = \frac{a}{h} = \frac{1}{\sqrt{2}}$

$\tan 45° = \frac{o}{a} = \frac{1}{1} = 1$

The following can also be used for Special angles

***h* = 2 units**

eg: $\sin 60° = \frac{o}{h} = \frac{\sqrt{3}}{2}$



Examples 3

Determine the following without using a calculator:

a. $\cos 0° + \cos^2 45°$

b. $\sin^2 60° + \cos^2 60°$

c. $\frac{\tan 45° \,.\, \sin 90°}{\cos 30°}$

Solutions:

a. $1 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 + \frac{1}{2} = 1\tfrac{1}{2}$

b. $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

c. $\frac{(1)(1)}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

Exercise 3

CAN YOU: Follow examples above and determine:

1. $\tan^2 30°$
2. $\sin 30° \times \cos 60°$
3. $\sin 90° + \cos 0° - 2\tan 45°$

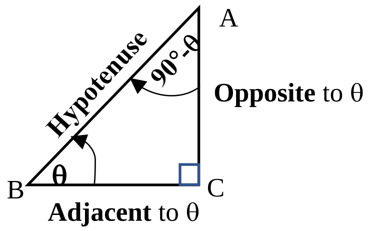
Solutions:

1. $\frac{1}{3}$ 2. $\frac{1}{4}$ 3. 0

Activities/ Assessment	Mind Action Series	Classroom Mathematics	Siyavula
	Ex 9 Numbers a, b and c Pg 85	Ex 5.2 Numbers 1 and 2 Ex 5.9 Numbers 1 and 2 Pg	Ex 7 – 1 Numbers 1, 4, 5 and 6 Pg.

Consolidation:

- Pythagoras for a right-angled ΔABC with $\hat{C} = 90°$: $c^2 = a^2 + b^2$
- Trigonometry is about the inter-relationship between the lengths of the sides and the sizes of the angles in a triangle.
- We use Greek letters (θ, α, β, etc.) to denote the angles in trigonometry.
- θ and (90°- θ) are a pair of complementary angles in a right-angled triangle.
- The side across the 90° angle (the longest side) is called the HYPOTENUSE (h)
- The side touching angle θ is ADJACENT (a) to θ.
- The side across angle θ is OPPOSITE (o) to θ.



- In any right-angled ΔABC we can write ratios in terms of the sides with respect to one-another:

$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

$$\tan\theta = \frac{o}{a}$$

- There are 6 ratios – the other 3 are called the reciprocals, but this 3 ratios form the basis of Trigonometry for Gr 10 – 12. **You must know them!!**
- Also know how to find trig. ratios of the special angles: 0°; 30°; 45°; 60° and 90°