SUBJECT and GRADE	MATHEMATICS GRADE 10	
TERM 2	Week 7	
TOPIC	TRIGONOMETRY	
AIMS OF LESSONS	*To:* • *Use calculator with trig ratios* • *Solve trig equations for angles between 0° and 90°* • *Solve triangles (lengths of sides/ size of angles)* • *Use diagrams to determine numeric values of ratios between 0° and 360°*	
RESOURCES	***Paper based resources***	***Digital resources***
	Textbook: Chapter on Trigonometry	https://video.tutonic.org/T10trigunknownsides https://youtu.be/zbB_SddM6Gs https://youtu.be/PG5sF-AeBX0 *Scientific calculator*
INTRODUCTION	*In the previous lesson you were introduced to Trigonometry where we looked at the definition of the trigonometric ratios in terms of a right-angled triangle as well as the ratios of special angles. In this lesson we continue with using the calculator and we define the trigonometric ratios in terms of the Cartesian plane.*	
CONCEPTS AND SKILLS	• *Using calculator* • *CAST diagram* • *Cartesian plane* • *Standard form of trig ratio/ eqaution*	

LESSON 1: USE CALCULATOR WITH TRIGONOMETRIC RATIOS

Examples

A. Evaluate with the use of a calculator and round off answers to 2 decimals:	**Solutions** and keystroke on calculator (Casio FX-82ZA)
1. $\sin 36°$	1. 0,59 
2. $\frac{\cos 24°}{2}$	2. 0,46 
3. $\tan^2 125°$	3. 2,04 
4. $\sin(46° + 27°)$	4. 0,96 
5. $\sin 46° + \sin 27°$	5. 1,17 

NOTE:

- $\sin(46° + 27°) \neq \sin 46° + \sin 27°$
- $2 \sin 37° \neq \sin(2 \times 37°)$
- $\sin 24° + 1 \neq \sin 25°$

Can you do the following?

Determine with a calculator, rounded to 2 decimals:

1. $\tan 76°$
2. $\frac{\sin 76°}{\cos 76°}$
3. $\cos^2 125° + \sin^2 125°$
4. $\frac{\sin 127°.\tan 323° + 2\cos 42°}{8 \sin 21°}$

Solutions: 1. 4, 01 2. 4, 01 3. 1 4. 0, 31

LESSON 2: Calculate angles using calculator/ **TRIG EQAUTIONS**

Examples

B. Calculate the size of the following angles, rounded off to 2 decimal places:

1. $\tan P = 0{,}684$
2. $\sin 3A = 0{,}821$
3. $2\cos(\theta - 10°) = 1{,}268$

Can't divide by 3 from start

Standard form of trig equation:

Trig ratio (angle) = value

Solutions and keystroke on calculator (Casio FX-82ZA)

1. $\angle P = \tan^{-1}(0{,}684) = 34{,}37°$ SHIFT tan 0 • 6 8 4) =

arctan

2. $3A = \sin^{-1}(0{,}821) = 55{,}19°$ SHIFT sin 0 • 8 2 1) =

$\therefore \angle A = \frac{55.19}{3} = 18{,}40°$

3. $\cos(\theta - 10°) = \frac{1{,}268}{2} = 0{,}634$

Can't add 10° here

$\therefore (\theta - 10°) = \cos^{-1} 0{,}634 = 50{,}65°$

$\therefore \quad \theta = 50{,}65° + 10°$

Only Now!

$= 60{,}65°$

Can you do the following?

Calculate te following angles and round off answers to 2 decimals:

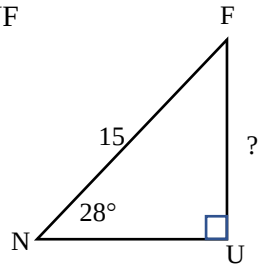
1. $\sin A = 0{,}234$
2. $2\tan\theta = 1{,}372$
3. $\frac{1}{4}\sin(2A + 20°) = 0{,}214$

Answer: 1. A = 13, 53° 2. θ = 34, 45° 3. A = 19, 44°

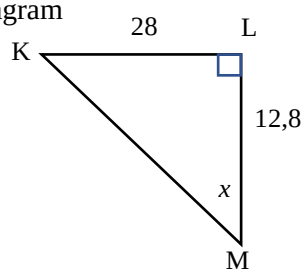
LESSON 3: SOLVING TRIANGLES

Examples

1. In ΔFUN, FN = 15, ∠U = 90° and ∠N = 28°. Calculate the length of UF



2. Calculate the size of angle x in the diagram



Solutions:

1. Side UF is **Opposite** to 28° and FN is the **Hypotenuse**

Write ratio with UF as numerator

$$\Rightarrow \frac{\text{UF}}{15} = \frac{o}{h} = \sin 28°$$

$$\therefore \text{UF} = 15 \sin 28°$$

1 5 sin 2 8) =

$$= 7{,}04$$

2. 28 is **opposite** to x and 12,8 is **adjacent** to x

$$\Rightarrow \tan x = \frac{o}{a} = \frac{28}{12{,}8} = 2{,}1875$$

Choose **tan** x since **opp** and **adj** sides are given

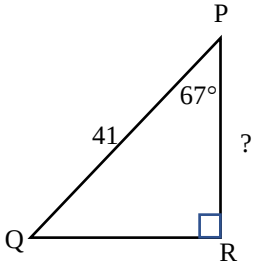
$$\therefore x = \tan^{-1}(2{,}1875)$$

$$= 65{,}43°$$

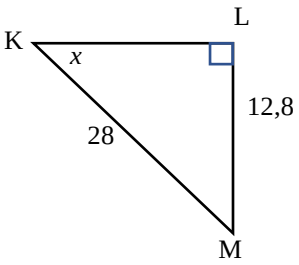
Don't round off here!!

Can you do the following?

1. In ΔPQR, PQ = 41, ∠R = 90° and ∠P = 67°. Calculate the length of PR.



2. Calculate the size of angle x in the diagram

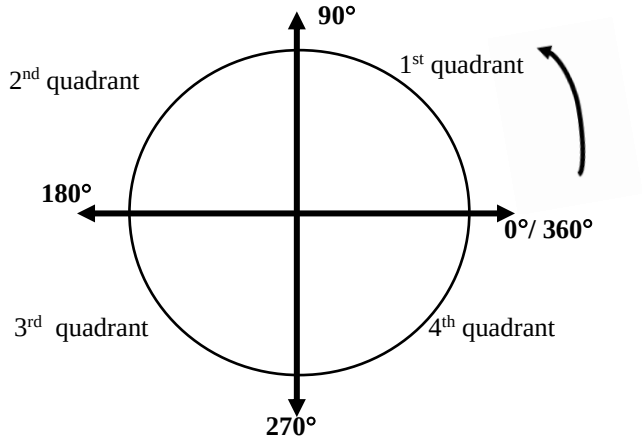


Answer: 1. PR = 16, 02

2. x = 27, 20°

LESSON 4: DEFINE TRIG RATIOS IN TERMS OF THE CARTESIAN PLANE

The Cartesian Plane for Trigonometry: The different quadrants

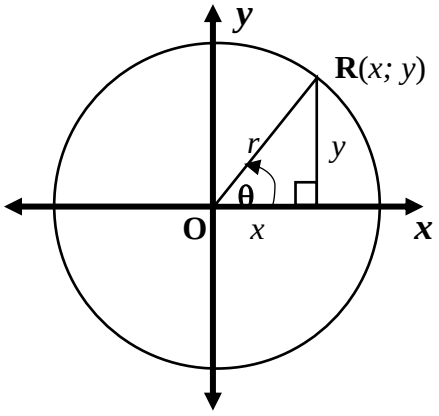


- The Cartesian plane are divided into 4 quadrants with intervals of 90°.
- We measure angles in an anti-clockwise way, starting with 0° on the positive horizontal axis.
  - 1st quadrant is between 0° and 90°
  - 2nd quadrant is between 90° and 180°
  - 3rd quadrant is between 180° and 270° and
  - 4th quadrant is between 270° and 360°
- Note: 0° and 360° (a revolution) are at the same point (starting/ ending points are the same)

CAN YOU say in which quadrants will the following angles lie?

1. 39° 2. 132° 3. 346°
4. 101° 5. 271° 6. 89°

How to draw sketches (triangles) in the Cartesian Plane

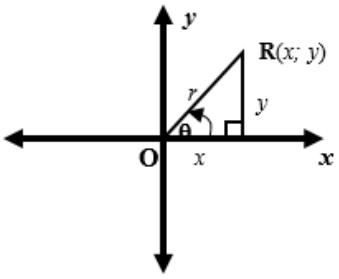


See the sketch:

- We use the coordinates of the point R$(x;\ y)$.
- Draw a line from the origin $(0;\ 0)$ to the point. That distance, OR, is called the terminal arm or r, the radius (of the circle through R) – the circle need not be drawn.
- Now draw a $\perp$ line from R on the $x-$ axis to form a right-angled triangle.
- The angle that OR makes with the positive $x-$ axis is θ - always measured in an anti-clockwise direction – in this case $0° < \theta < 90°$, hence the sketch is in the **1st quadrant**.

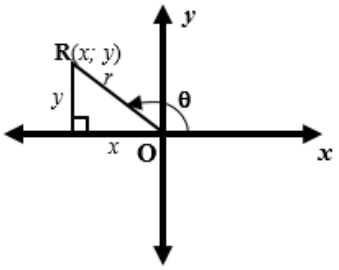
Triangles in the different quadrants: Note: *how do we show the angle θ in the different quadrants*

1st quadrant: $0° < \theta < 90°$



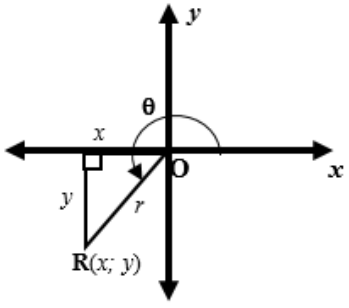
x	$+$
y	$+$
r	$+$
$\sin\theta$	$\frac{y}{r}=\frac{+}{+}=+$
$\cos\theta$	$\frac{x}{r}=\frac{+}{+}=+$
$\tan\theta$	$\frac{y}{x}=\frac{+}{+}=+$

2nd quadrant: $90° < \theta < 180°$



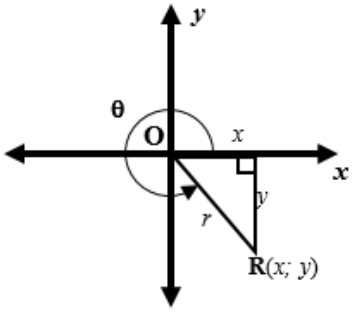
x	$-$
y	$+$
r	$+$
$\sin\theta$	$\frac{y}{r}=\frac{+}{+}=+$
$\cos\theta$	$\frac{x}{r}=\frac{-}{+}=-$
$\tan\theta$	$\frac{y}{x}=\frac{+}{-}=-$

3rd quadrant: $180° < \theta < 270°$



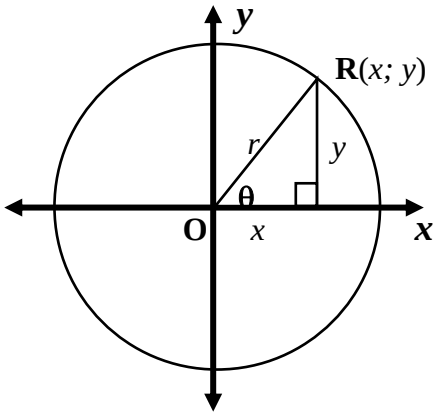
x	$-$
y	$-$
r	$+$
$\sin\theta$	$\frac{y}{r}=\frac{-}{+}=-$
$\cos\theta$	$\frac{x}{r}=\frac{-}{+}=-$
$\tan\theta$	$\frac{y}{x}=\frac{-}{-}=+$

4th quadrant: $270° < \theta < 360°$



x	$+$
y	$-$
r	$+$
$\sin\theta$	$\frac{y}{r}=\frac{-}{+}=-$
$\cos\theta$	$\frac{x}{r}=\frac{+}{+}=+$
$\tan\theta$	$\frac{y}{x}=\frac{-}{+}=-$

The Trigonometric ratios in the Cartesian plane



If we take any point R$(x;\ y)$ in the Cartesian plane, it is noted that:

$$\sin\theta = \frac{o}{h} = \frac{y}{r}$$

$$\cos\theta = \frac{a}{h} = \frac{x}{r}$$

$$\tan\theta = \frac{o}{a} = \frac{y}{x}$$

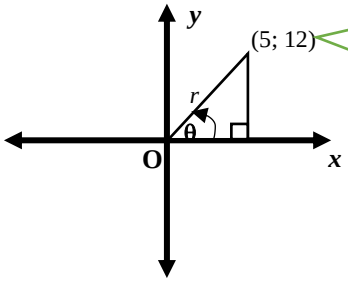
Examples

1. Use the diagram to determine the value of:

(a) r

(b) $\sin\theta$

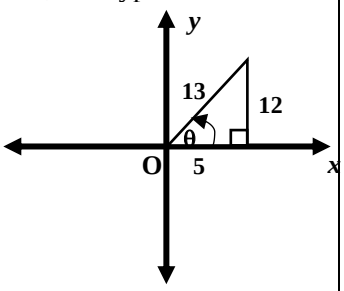
(c) $\tan^2\theta$



This is the coordinates
$(x;\ y) \Rightarrow x = 5$ and $y = 12$

Solutions:

(a) Write 5 along the x −axis and 12 along the y −axis
Use Pythagoras to calculate r, the hypotenuse



$r^2 = x^2 + y^2$… Pyth.
$= (5)^2 + (12)^2$
$= 25 + 144 = 169$
$\therefore r = \sqrt{169}$
$\therefore r = 13$

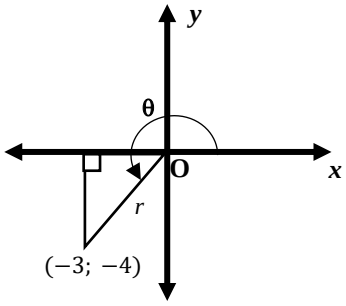
(b) $\sin\theta = \frac{y}{r} = \frac{12}{13}$

(c) $\tan^2\theta = \left(\frac{y}{x}\right)^2 = \left(\frac{12}{5}\right)^2 = \frac{144}{25} = 5\frac{19}{25}$

Can you use the diagram to determine the value of:

(a) r

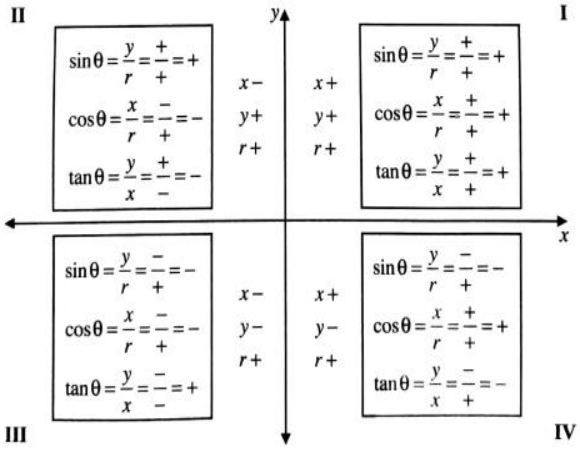
(b) $25\sin^2\theta - 5\cos\theta$



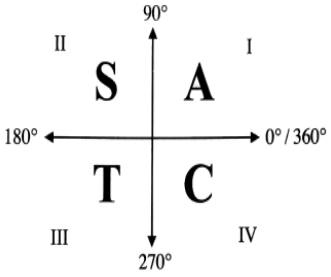
Solutions: (a) r = 5 (b) 19

LESSON 5: Signs Of Trigonometric Ratios In The Quadrants

The following diagram shows the signs of x, y and r, as well as the signs of the three trigonometric ratios in each of the four quadrants.



We summarise this in the so-called CAST diagram



The letters A, S, T and C indicate which ratio(s) are **positive** in each quadrant.

- In quadrant I: **A**ll trig ratios are positive
- In quadrant II: **S**in is **positive** and all other ratios are negative
- In quadrant III: **T**an is **positive** and all other ratios are negative
- In quadrant IV: **C**os is **positive** and all other ratios are negative

The so- called **CAST** diagram; also referred to as:
All **S**tations **T**o **C**apetown

Examples

1. In which quadrant(s) are:
   a. tan θ positive
   b. sin θ < 0 and cos θ < 0
   c. tanθ > 0 and sin θ > 0
   d. cos θ < 0 and 180° < θ < 0°

Solutions:

1 a. quadrants 1 and 3
b. quadrant 3
c. quadrant 1
d. quadrant 2

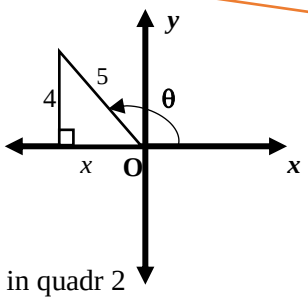
Trigonometric ratios in the different quadrants

1. If $5\sin\theta - 4 = 0$ and $\theta \in [90°; 270°]$, determine, without using a calculator, the value of $\cos^2\theta$

Solution: $5\sin\theta - 4 = 0$

$\therefore \sin\theta = +\frac{4}{5} = \frac{y}{r}$

Draw a sketch in quadrant 2



$\Rightarrow y = 4$ and $r = 5$

Using Pyth: $r^2 = x^2 + y^2$

$\therefore x^2 = r^2 - y^2$

$= 25 - 16$

$= 9$

$\therefore x = \pm 3$

$\Rightarrow x = -3$ since x is negative in quadr 2

$\therefore \cos^2\theta = \left(\frac{x}{r}\right)^2 = \left(\frac{-3}{5}\right)^2 = \frac{9}{25}$

NOTE:
Without using a calculator means**: using a sketch!!**

Write the trig equation in standard form:
trig ratio (angle) = number

Ask yourself:
In which quadrants is sin θ positive (+)?
In quadrants 1 and 2
Which quadrants are included in the given interval ?
Quadrants 2 and 3
So, angle θ has to be in quadrant 2, as it is the quadrant which is in the specified interval as well as where sin is positive.

Can you do the following?

If $41\cos\theta + 9 = 0$ and $180° < \theta < 360°$, determine, without the use of a calculator the value of:
$9\tan\theta + 41\sin\theta$

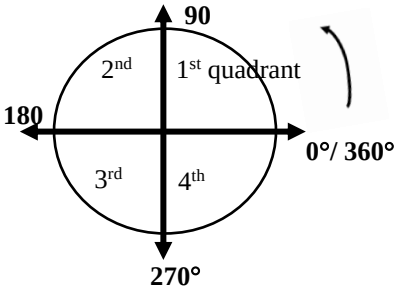
Solution: 80

ACTIVITIES:

Siyavula	Mind Action Series	Classroom Mathematics	Via Africa
Ex: 7 -1 No. 2 and 3 Pg 225 Ex: 7 – 4 No 1 and 2 Pg 232	Ex: 2 pg 68 Ex: 3 pg 70 Ex: 5 pg 75 Ex: 8 pg 82	Ex: 5.3 ; No 1 and 2 Pg 110 Ex: 5.7 ; No 1 and 2 Pg 118 Ex 5.8 Pg 121	Ex: 13; Pg: 194

CONSOLIDATION:

- Know how to use keys on the calculator to determine trig ratios and how to calculate the angles.
- The Cartesian plane are divided into 4 quadrants with intervals of 90°.
- We measure angles in an **anti-clockwise** way, starting with 0° on the positive horizontal axis.

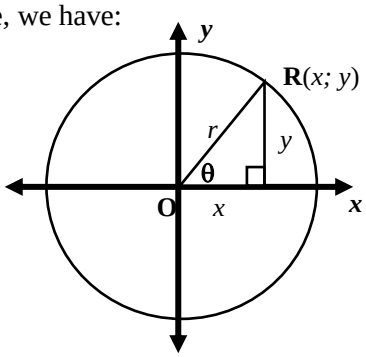


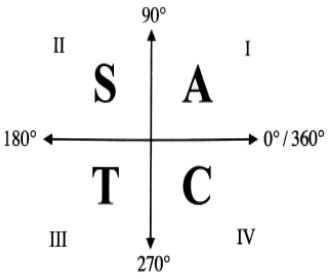
- In the Cartesian plane, we have:

$\sin\theta = \frac{o}{h} = \frac{y}{r}$

$\cos\theta = \frac{a}{h} = \frac{x}{r}$

$\tan\theta = \frac{o}{a} = \frac{y}{x}$





- We determine the signs of the trig ratios in the quadrants by using the so- called **CAST** diagram; also referred to as:
  "**A**ll **S**tations **T**o **C**apetown"
  The letters A, S, T and C indicate which ratio(s) are **positive** in each quadrant.
- To solve trig ratios/ equations in the quadrants (without using a calculator): ⇒ **using a sketch**
  - Write the equation in standard form: **trig ratio (angle) = number**
  - Determine in which quadrant the angle lies by usig the CAST diagram
  - Draw a sketch (right-angled triangle) in the quadrant and fill in the values for x, y and/or r
  - Calculate the missing side using Pythagoras
  - Determine the asked ratio(s)