

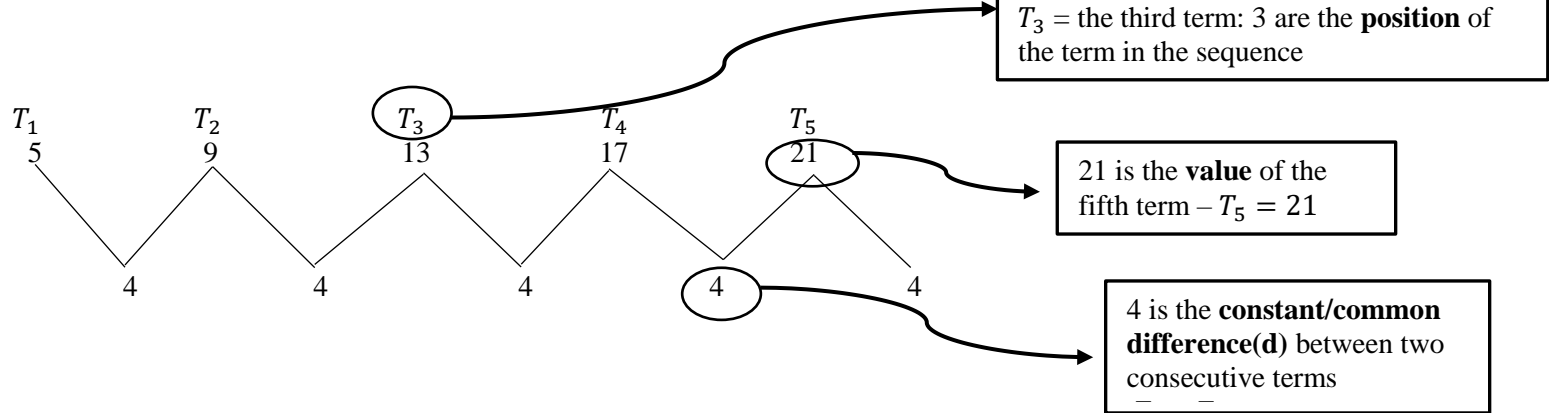


SUBJECT and GRADE	MATHEMATICS GRADE 10																																																																		
TERM 3	Week 4																																																																		
TOPIC	PATTERNS																																																																		
AIMS OF LESSON	Recognise a linear pattern. Find the general term of the pattern Find a term in the given position of a linear pattern																																																																		
RESOURCES	<i>Paper based resources</i>	<i>Digital resources</i>																																																																	
	Textbook chapter on Patterns	<a href="https://www.youtube.com/watch?v=V02nV_qR_xQ">https://www.youtube.com/watch?v=V02nV_qR_xQ</a> <a href="https://www.youtube.com/watch?v=Yd80NCXBINU">https://www.youtube.com/watch?v=Yd80NCXBINU</a>																																																																	
INTRODUCTION	In the previous grades you were introduced to numeric number patterns which is a sequence of numbers that follow a specific pattern. An example of a <b>linear pattern</b> is one where there is a <b>constant difference between consecutive terms</b> . In other words, the <b>same number</b> will be added to, or subtracted from each consecutive term.																																																																		
Look at the following sequences:																																																																			
<p>Example 1.</p> <p>2; 4; 6; 8; ...</p> <table border="1"> <thead> <tr> <th><math>n</math></th> <th><math>T_n</math> : general term</th> </tr> </thead> <tbody> <tr> <td>1</td> <td><math>2 = 2(1) = 2</math></td> </tr> <tr> <td>2</td> <td><math>2+2 = 2(2) = 4</math></td> </tr> <tr> <td>3</td> <td><math>2+2+2 = 2(3) = 6</math></td> </tr> <tr> <td>4</td> <td><math>2+2+2+2 = 2(4) = 8</math></td> </tr> <tr> <td>10</td> <td><math>= 2(10) = 20</math></td> </tr> <tr> <td>100</td> <td><math>= 2(100) = 200</math></td> </tr> <tr> <td><math>n</math></td> <td><math>= 2(n)</math></td> </tr> </tbody> </table> <p><b>Multiples of 2 = <math>2(n)</math></b> <b>Even numbers</b></p> <p><b>Odd numbers = <math>2n - 1</math></b></p>	$n$	$T_n$ : general term	1	$2 = 2(1) = 2$	2	$2+2 = 2(2) = 4$	3	$2+2+2 = 2(3) = 6$	4	$2+2+2+2 = 2(4) = 8$	10	$= 2(10) = 20$	100	$= 2(100) = 200$	$n$	$= 2(n)$	<p>Example 2.</p> <p>3; 6; 9; 12; ...</p> <table border="1"> <thead> <tr> <th><math>n</math></th> <th><math>T_n</math> : general term</th> </tr> </thead> <tbody> <tr> <td>1</td> <td><math>3 = 3(1) = 3</math></td> </tr> <tr> <td>2</td> <td><math>3+3 = 3(2) = 6</math></td> </tr> <tr> <td>3</td> <td><math>3+3+3 = 3(3) = 9</math></td> </tr> <tr> <td>4</td> <td><math>3+3+3+3 = 3(4) = 12</math></td> </tr> <tr> <td>10</td> <td><math>= 3(10) = 30</math></td> </tr> <tr> <td>100</td> <td><math>= 3(100) = 300</math></td> </tr> <tr> <td><math>n</math></td> <td><math>= 3(n)</math></td> </tr> </tbody> </table> <p><b>Multiples of 3 = <math>3(n)</math></b></p>	$n$	$T_n$ : general term	1	$3 = 3(1) = 3$	2	$3+3 = 3(2) = 6$	3	$3+3+3 = 3(3) = 9$	4	$3+3+3+3 = 3(4) = 12$	10	$= 3(10) = 30$	100	$= 3(100) = 300$	$n$	$= 3(n)$	<p>Example 3.</p> <p>2; 4; 8; 16; ...</p> <table border="1"> <thead> <tr> <th><math>n</math></th> <th><math>T_n</math> : general term</th> </tr> </thead> <tbody> <tr> <td>1</td> <td><math>2 = 2^1 = 2</math></td> </tr> <tr> <td>2</td> <td><math>2 \times 2 = 2^2 = 4</math></td> </tr> <tr> <td>3</td> <td><math>2 \times 2 \times 2 = 2^3 = 8</math></td> </tr> <tr> <td>4</td> <td><math>2 \times 2 \times 2 \times 2 = 2^4 = 16</math></td> </tr> <tr> <td>10</td> <td><math>= 2^{10} = 1024</math></td> </tr> <tr> <td>100</td> <td><math>= 2^{100}</math></td> </tr> <tr> <td><math>n</math></td> <td><math>= 2^n</math></td> </tr> </tbody> </table> <p><b>Powers of 2 = <math>2^n</math></b></p>	$n$	$T_n$ : general term	1	$2 = 2^1 = 2$	2	$2 \times 2 = 2^2 = 4$	3	$2 \times 2 \times 2 = 2^3 = 8$	4	$2 \times 2 \times 2 \times 2 = 2^4 = 16$	10	$= 2^{10} = 1024$	100	$= 2^{100}$	$n$	$= 2^n$	<p>Example 4.</p> <p>1; 4; 9; 16; ...</p> <table border="1"> <thead> <tr> <th><math>n</math></th> <th><math>T_n</math> : general term</th> </tr> </thead> <tbody> <tr> <td>1</td> <td><math>1 \times 1 = 1^2</math></td> </tr> <tr> <td>2</td> <td><math>2 \times 2 = 2^2 = 4</math></td> </tr> <tr> <td>3</td> <td><math>3 \times 3 = 3^2 = 9</math></td> </tr> <tr> <td>4</td> <td><math>4 \times 4 = 4^2 = 16</math></td> </tr> <tr> <td>10</td> <td><math>= 10^2 = 100</math></td> </tr> <tr> <td>100</td> <td><math>= 100^2 = 10000</math></td> </tr> <tr> <td><math>n</math></td> <td><math>= n^2</math></td> </tr> </tbody> </table> <p><b>Squares of natural numbers = <math>n^2</math></b></p> <p><b>Cubes of natural numbers = <math>n^3</math></b></p>	$n$	$T_n$ : general term	1	$1 \times 1 = 1^2$	2	$2 \times 2 = 2^2 = 4$	3	$3 \times 3 = 3^2 = 9$	4	$4 \times 4 = 4^2 = 16$	10	$= 10^2 = 100$	100	$= 100^2 = 10000$	$n$	$= n^2$
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**Linear Patterns:**

An example of a **linear pattern** is one where there is a **constant difference between consecutive terms**. In other words, the **same number** will be added to, or subtracted from each consecutive term.

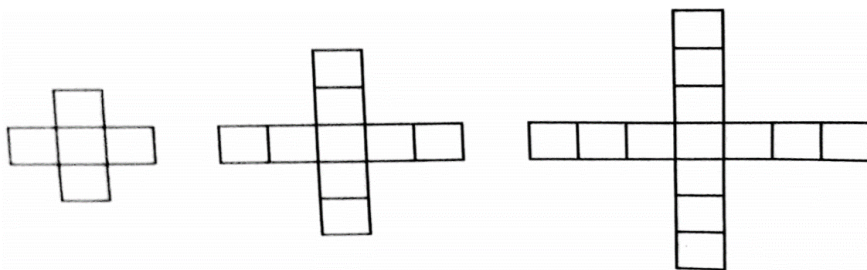
Example 5. Consider the **pattern/ sequence: 5; 9; 13; 17; 21; ...**



**The sequence can be described in words as follows:** The 1<sup>st</sup> term is 5. Add 4 to the 1<sup>st</sup> term to obtain the 2<sup>nd</sup> term which is 9. Continue adding 4 to each term to obtain the next term.

- The constant difference is 4
- The next three terms is 25; 29; 33
- The value of  $T_4 = 17 \rightarrow 17$  is the term in the 4<sup>th</sup> position
- The 2<sup>nd</sup> term  $\rightarrow T_2 = 9$  is the value of 2<sup>nd</sup> term

Example 6: Consider the **pattern** of squares:



We can represent the pattern of squares on a table.

**Constant difference**  $d = T_2 - T_1 = 9 - 5 = 4$

$n$	Number of squares	$T_n$ : general term
1	$5 = 5$	$= 5 + 4(0)$
2	$5 + 4 = 9$	$= 5 + 4(1)$
3	$5 + 4 + 4 = 13$	$= 5 + 4(2)$
4	$5 + 4 + 4 + 4 = 17$	$= 5 + 4(3)$
$n$	$5 + 4 + 4 + 4 + 4 + \dots + 4$	$= 5 + 4(n - 1)$

<p><b>Example 7:</b> Given the pattern: -2; 1; 4; 7</p> <p>a) Determine the general rule to determine any term in the pattern.</p> <p>b) Use the general rule to determine the 50<sup>th</sup> term in the pattern.</p> <p>c) Which term in the pattern will be equal to 55.</p>	<p><b>Solution:</b> <b>Constant difference</b> <math>d = T_2 - T_1 = 1 - (-2) = 3</math></p> <table border="1" data-bbox="621 282 1222 597"> <thead> <tr> <th><math>n</math></th> <th colspan="2"><math>T_n</math> : general term</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>= -2</td> <td>= -2+3(0)</td> </tr> <tr> <td>2</td> <td>-2+ 3</td> <td>= 1</td> <td>= -2+3(1)</td> </tr> <tr> <td>3</td> <td>-2+3+3</td> <td>= 4</td> <td>= -2+3(2)</td> </tr> <tr> <td>4</td> <td>-2+3+3+3</td> <td>= 7</td> <td>= -2+3(3)</td> </tr> <tr> <td><math>n</math></td> <td>-2+3+3+3+ ... +3</td> <td></td> <td>= -2+3(<math>n - 1</math>) = -2 + 3<math>n</math> - 3 = <b>3<math>n</math> - 5</b></td> </tr> </tbody> </table> <p style="text-align: center;"><math>\therefore T_n = 3n - 5</math></p> <p><math>T_n = 3n - 5</math>  <math>\therefore T_{50} = 3(50) - 5</math>  <math>T_{50} = 145</math></p> <div style="border: 1px solid black; padding: 5px; display: inline-block;">The <b>position</b> of the term is 50. Therefore, <math>n = 50</math></div> <p><math>T_n = 55</math>  <math>\therefore T_n = 3n - 5 = 55</math>  <math>3n = 55 + 5</math>  <math>3n = 60</math>  <math>n = 20</math></p> <p><math>\therefore</math> 20<sup>th</sup> term of the pattern is equal to 55</p>	$n$	$T_n$ : general term		1	-2	= -2	= -2+3(0)	2	-2+ 3	= 1	= -2+3(1)	3	-2+3+3	= 4	= -2+3(2)	4	-2+3+3+3	= 7	= -2+3(3)	$n$	-2+3+3+3+ ... +3		= -2+3( $n - 1$ ) = -2 + 3 $n$ - 3 = <b>3<math>n</math> - 5</b>	<p><b>CAN YOU?</b></p> <p>Given the following sequence: 3; 8; 13; 18; ...</p> <p>Determine:</p> <p>a) The general term</p> <p>b) The 20<sup>th</sup> term</p> <p>c) Which term of the sequence is equal to 223</p> <p><b>Answers:</b></p> <p>a) <math>T_n = 5n - 2</math></p> <p>b) 98</p> <p>c) <math>n = 45</math></p>
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<p><b>Example 8:</b> Write down the first three terms and the constant difference for a linear pattern with general term: <math>T_n = 6n + 5</math></p>	<p><b>Solution:</b> First 3 terms  <math>T_1 = 6(1) + 5 = 11</math>  <math>T_2 = 6(2) + 5 = 17</math>  <math>T_3 = 6(3) + 5 = 23</math></p>	<p>Constant difference:  <math>d = T_2 - T_1</math> or <math>d = T_3 - T_2</math>  <math>d = 17 - 11</math>  <math>d = 6</math></p>																							
<p>ACTIVITIES/ASSESSMENT</p>	<table border="1"> <thead> <tr> <th colspan="2">Mind Action Series</th> </tr> <tr> <th>Exerise</th> <th>Page</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>57</td> </tr> </tbody> </table>	Mind Action Series		Exerise	Page	1	57	<table border="1"> <thead> <tr> <th colspan="2">Siyavula</th> </tr> <tr> <th>Exerise</th> <th>Page</th> </tr> </thead> <tbody> <tr> <td>3.1</td> <td>64</td> </tr> </tbody> </table>	Siyavula		Exerise	Page	3.1	64	<table border="1"> <thead> <tr> <th colspan="2">Classroom Mathematics</th> </tr> <tr> <th>Exerise</th> <th>Page</th> </tr> </thead> <tbody> <tr> <td>3.1, 3.2, 3.3</td> <td>59, 61, 64</td> </tr> </tbody> </table>	Classroom Mathematics		Exerise	Page	3.1, 3.2, 3.3	59, 61, 64				
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<p>CONSOLIDATION</p>	<ul style="list-style-type: none"> <li>Linear patterns have a constant/common difference (<math>d</math>)</li> <li>Find the general term/ <math>n</math>th term:</li> <li>Find a term in the given position of a linear pattern</li> </ul>																								