SUBJECT and GRADE	Mathematics Gr 11	
TERM 3	Week 7	
TOPIC	Trigonometry: Formulae to solve triangles	
AIMS OF LESSON	To apply formulae to calculate the sizes of angles and lengths of sides related to Problems in 2D.	
RESOURCES	***Paper based resources***	***Digital resources***
	Please refer to the section on the Solution of Triangles/ Trigonometry in solving triangles and then to Solving Problems in 2D in your Textbook.	https://www.youtube.com/watch?v=j3VLbjsWdHo
INTRODUCTION	• In the previous lesson we looked at the Area rule and mixed examples to calculate the unknown sides and/or angles as well as the Area of any triangle. • In this lesson we will look at Solving problems in 2 Dimensions where we apply the rules.	
CONCEPTS AND SKILLS	• Using Theorem of Pythagoras • Solving right-angled triangles • Using the Area/ sin/cos rules • Using the trig ratios: sin θ, cos θ en tan θ • Using the scientific calculator to do trigonometric calculations	

Lesson 1: Applying rules on Problems in 2D

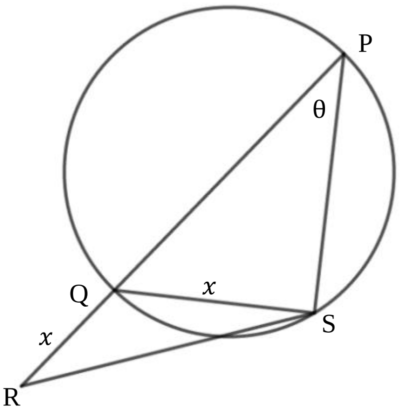
1. In the diagram, PQ is the diameter of the circle with centre O. PQR is a straight line and QR = QS = x.

(a) Prove that $RS^2 = 2x^2(1 + \sin\theta)$

(b) If $RS = \sqrt{12}$ and $x = 2$

(i) Show, without using a calculator, that $\theta = 30°$ and PQ = 4

(ii) Calculate the Area of ΔRQS



Solution:

(a) In ΔPQS: $Q\hat{S}P = 90°$…. ∠ in semi-circle

$\therefore R\hat{Q}S = 90° + \theta$ …exterior ∠ of Δ

Now: In ΔRQS: $\mathbf{RS^2} = QS^2 + QR^2 - 2.QS.QR.\cos(90° + \theta)$

Cos rule for ΔRQS

$= x^2 + x^2 - 2.x.x.(-\sin\theta)$

$= 2x^2 + 2x^2\sin\theta$

$= \mathbf{2x^2(1 + \sin\theta)}$

(b) (i) $RS^2 = 2x^2(1 + \sin\theta)$

$\therefore (\sqrt{12})^2 = 2(2)^2(1 + \sin\theta)$

$\therefore 12 = 8(1 + \sin\theta)$

$\therefore 1{,}5 = 1 + \sin\theta$

$\therefore \sin\theta = 0{,}5$

$\therefore \boldsymbol{\theta = 30°}$

(i) $\sin\theta = \frac{QS}{PQ}$

$\therefore 0{,}5 = \frac{2}{PQ}$

$\therefore \mathbf{PQ = 4}$

(ii) Area $\Delta RQS = \frac{1}{2}RQ.QS.\sin(90° + \theta)$

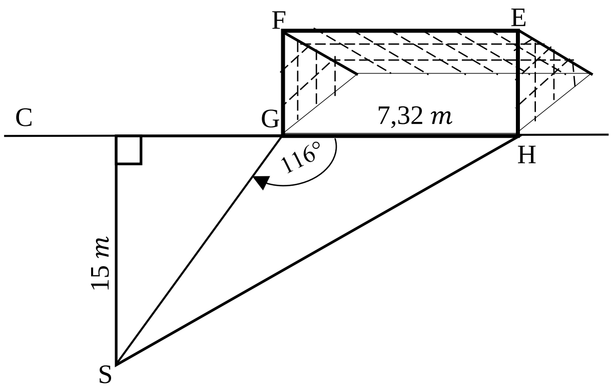
$= \frac{1}{2}(2)(2)\sin(90° + 30°)$

$= \sqrt{\mathbf{3}}$ **(units)^2**

Can you do?

2. A soccer player aims towards the goal which is 15 metres from the back line CH on a soccer field. The angle from the left goal post, FG to the soccer player, S is 116°. The goal posts are 7, 32 m wide. The diagram represents the above situation. Calculate:

2.1 The size of $C\hat{G}S$
2.2 How far the soccer player is from the left goal post, FG (calculate the distance GS)
2.3 How far the soccer player is from the right goal post, EH
2.4 The approximate size of $G\hat{S}H$, the angle within which the soccer player could possibly score a goal

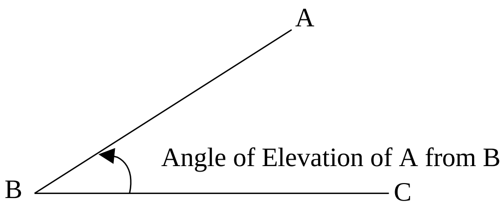


Solutions:

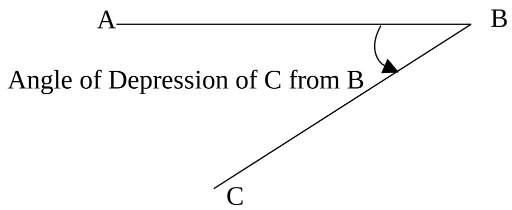
1.1 $C\hat{G}S = 64°$
1.2 $GS = 16{,}9\ m$
1.3 $SH = 20{,}96\ m$
1.4 $G\hat{S}H = 18{,}29°$

Lesson 2: Angles of Elevation/ Depression

Angle of Elevation (from horizontal upwards)



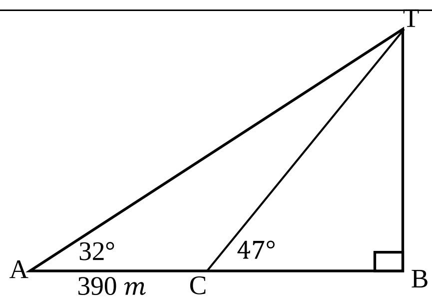
Angle of Depression (from horizontal downwards)



Example:

1. The diagram represents a vertical tower TB. A person standing at a point A, in the same horizontal plane as the base of the tower observes that the angle of elevation to T, the top of the tower is 32°. Another person standing at point C, 390 m from the person at A observes the angle of elevation to T is 47°. ACB is in a straight line. Calculate:

1.1 the size of $A\hat{T}C$
1.2 the distance from the top of the tower to the man standing at C, TC
1.3 the height of the tower, TB



Solution:

1.1 $A\hat{T}C = 15°$ ... exterior ∠ of Δ
1.2 **In ΔATC**: [2 angles + 1side opp an angle given] ⇒ **use sin-rule**

$$\frac{TC}{\sin A} = \frac{AC}{\sin A\hat{T}C} \quad \therefore \quad \frac{TC}{\sin 32°} = \frac{390}{\sin 15°}$$

$$\therefore \ TC = \frac{390 \sin 32°}{\sin 15°} = 798,51\ m$$

1.3 ΔTBC is right-angled ⇒use trig ratios

$$\frac{TB}{CT} = \sin T\hat{C}B$$

$$\therefore \frac{TB}{789{,}51} = \sin 47°$$

$$\therefore TB = 789{,}51 \sin 47°$$

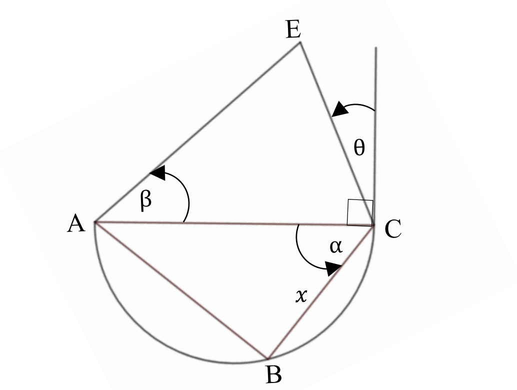
$$= 583{,}99\ m$$

Note: Calculations/ Proofs when working with problems in 2D

- Usually, 2 triangles with a common/ connecting side will be given; one that is right-angled and the other one that is scalene – identify them.
- In the right-angled triangle, we use the 3 trig ratios (sin θ; cos θ; tan θ) to calculate sides/ angles or write sides in terms of another.
- In the scalene triangle, we use the sin/ cos-rule to calculate sides/ angles.
- Also note that Euclidean Geometry may sometimes be needed to calculate the size of angles.
- Start with the triangle that has the most given information and calculate the common side depending on the type of triangle, otherwise start with what was asked.
- Usually this common side will provide a link to the follow-on question/ required answer.
- Only use Area rule if asked to calculate Area.

Example:

In the diagram, AC is the diameter of the semi-circle. The angle of elevation of E from A is β and the angle of depression of B from C is α. DC ⊥ AC with $E\hat{C}D = \theta$ and BC = x



1.1. What is the size of $A\hat{B}C$?

1.2. Show that AC = $\frac{x}{\cos\alpha}$

1.3. Determine an expression for $A\hat{E}C$ in terms of θ and β

1.4. Hence, prove that EC = $\frac{x.\sin\beta}{\cos\alpha.\cos(\beta-\theta)}$

Solution:

1.1. $A\hat{B}C = 90°$ … ∠ in semi-circle

1.2. $\frac{x}{AC} = \cos\alpha$

Use trig ratios in ΔABC since it is right-angled

$\Rightarrow AC = \frac{x}{\cos\alpha}$

AC is the common side!! Will be used again later

1.3. $E\hat{C}A = 90° - \theta$

$\therefore A\hat{E}C = 180° - \beta - (90° - \theta)$ … ∑ interior ∠s of ΔAEC

$= 90° - \beta + \theta = \mathbf{90° - (\beta - \theta)}$

Since we see (β − θ) in 1.4

1.4. $\frac{EC}{\sin\beta} = \frac{AC}{\sin[90°-(\beta-\theta)]}$

Use sin-rule in ΔAEC

$\therefore \frac{EC}{\sin\beta} = \frac{\left(\frac{x}{\cos\alpha}\right)}{\sin[90°-(\beta-\theta)]}$

Substitute the value of AC from 1.2

$= \frac{x}{\cos\alpha.\cos(\beta-\theta)}$

sin[90° − (β − θ)] = sin(β − θ)

$\therefore EC = \frac{x.\sin\beta}{\cos\alpha.\cos(\beta-\theta)}$

Can you do?

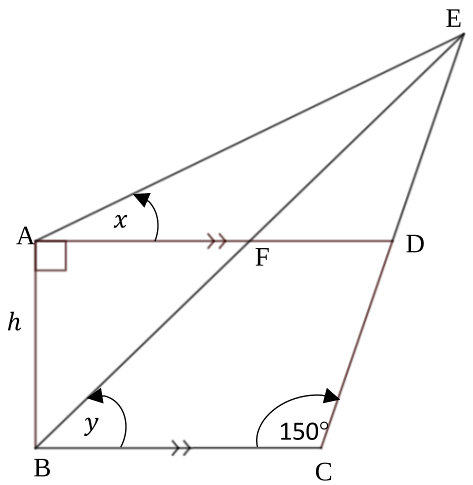
2. ABCD is a trapezium in the diagram with AD // BC. AD ⊥ AB and B$\hat{C}$D = 150°. CD is produced to E and BE and AE drawn. The angle of elevation of E from A is x and the angle of elevation of E from B is y. Let AB = h

2.1 Determine, with reasons, the size of A$\hat{E}$B.

2.2 Show that BE = $\dfrac{h\cos x}{\sin(y-x)}$

2.3 Hence prove that CE = $\dfrac{2h.\cos x.\sin y}{\sin(y-x)}$

2.4 If $h = 5\ m$, $x = 55°$ and $y = 70°$, calculate the length of CE.



Solution:
2.1 A$\hat{E}$B = $y - x$
2.2 use sin-rule in ΔABE
2.3 use sin-rule in ΔEBC
2.4 CE = 20,82 m

ACTIVITIES/ ASSESSMENT	*Please do the exercises as they appear in you textbook on* **Problems in 2D**

CONSOLIDATION:

- Usually, 2 triangles with a common/ connecting side will be given; one that is right-angled and the other one that is scalene – identify them.
- In the right-angled triangle, we use the 3 trig ratios (sin θ; cos θ; tan θ) to calculate sides/ angles or write sides in terms of another.
- In the scalene triangle, we use the sin/ cos-rule to calculate sides/ angles.
- Also note that Euclidean Geometry may sometimes be needed to calculate the size of angles.
- Start with the triangle that has the most given information and calculate the common side depending on the type of triangle, otherwise start with what was asked.
- Usually this common side will provide a link to the follow-on question/ required answer.
- Only use Area rule if asked to calculate Area.

- Thank you for participating in this lesson and please continue to work through the lessons as they are made available.
- Remember: Your hard work will reap success at the end!!

KEEP WORKING HARD !!