SUBJECT and GRADE	Mathematics Grade 10	
TERM 1	Week 2: Algebraic Expressions	
TOPIC	Factorisation	
AIMS OF LESSON	To revise factorisation started in Grade 9 and extend to Grade 10 factorisation	
RESOURCES	***Paper based resources***	***Digital resources***
	Please refer to the chapter in your textbook on Algebraic Expressions and Factorisation.	https://www.youtube.com/watch?v=5QyeZ7KwFKg https://www.youtube.com/watch?v=kAHRBxLhkW8 https://www.youtube.com/watch?v=FfhtmOKFra
INTRODUCTION	In this lesson we continue with the factorising of trinomials from grade 9. Factorisation is necessary for Algebra and several other sections of the curriculum, especially when drawing graphs. We will consider two different methods of factorising trinomials and proceed to factorize the sum and difference of cubes. We will end with a short test for you to consolidate your knowledge.	
CONCEPTS/ SKILLS	Factorisation is the "reverse" operation to Products or Expansion of monomials, binomials and trinomials Factorisation**:** Changing the sum expression (polynomial) to a product expression (monomial)	
Lesson 1	**Revision of Gr 9 Factorisation:** Difference of Squares and Quadratic Trinomials of the form $x^2 + bx + c$	

Grade 9: Revision from Week 1

1. Common Factor

This could be:
A negative sign, a number, variable or a bracket
Examples:

a) $(1-x)$ becomes $(-x+1) = -1(\boldsymbol{x}-\mathbf{1})$
b) $(2x-4) = 2(x-2)$
c) $x^3 - x = x(x^2-1)$
d) $(x-3)(x-2) - (2x+1)(x-2)$
$= (x-2)[x-3-(2x+1)]$
$= (x-2(x-3-2x-1)$
$= (x-2)(-x-4)$
$= -(x-2)(x+4)$

2. Factorising Difference of two Squares

Both terms must be squares; one positive and one negative term
i.e.
(x^2-y^2)
$= (x-y)(x+y)$

CAN YOU?

Exercise 1:

Factorise the following:

1. $5a^2 - 10b$
2. $4b^3 - 3b^2 - 5ab^2$
3. $\left(\frac{z^2}{4} - z - \frac{z^3}{2}\right)$
4. $(-2a + 10ab)$
5. $3a(2a-1) - 4(2a-1)$
6. $(x-y)^2 + (y-x)(x+y)$

Answers:1

1. $5(a^2-2b)$
2. $b(4b^2-3b-5ab)$
3. $\frac{z}{4}(z-4-2z^2)$
4. $2a(5b-1)$ or $-2a(1-5b)$
5. $(2a-1)(3a-4)$
6. $-2y(x-y)$

Exercise 2:

Factorise the following:

1. $5a^2 - 20$
2. $-3b^2 + 27$
3. $x^4 - 16$
4. $b^2 + 27$
5. $9b^2 - 25(c+d)^2$
6. $25w^2 - 16v^2 + (5w-4v)(2w+v)$

Answers: 2

1. $5(a^2-4 = 5(a-2)(a+2)$
2. $-3b^2+27 = -3(b^2-9)$
$= -3(b+3)(b-3)$
3. $x^4 - 16 = (x^2+4)(x^2-4)$
$= (x^2+4)(x-2)(x+2)$
4. $b^2+27 = b^2+27$ (sum of squares!)
5. $[3b-5(c+d)][3b+5(c+d)]$
6. $(5w-4v)(7w+5v)$

Lesson 2:	**Factorisation**: The trinomial: $x^2 + bx + c$

3. Factorising Trinomials of the form: $x^2 + bx + c$

Example 1: Factorise: $a^2 + \mathbf{5a} + 4$

Method 1: using Grouping

$$a^2 + \mathbf{5a} + 4$$
$$= a^2 + \mathbf{4a} + \boldsymbol{a} + 4$$
$$= (a^2 + 4a) + (a + 4)$$
$$= a(a + 4) + (a + 4)$$
$$= (a + 4)(a + 1)$$

Choose the **factors** of $+$ **4** such that if we add them, we get $+\mathbf{5}$, thus $(+4)$ and $(+1)$: $(+4)(+1) = +\mathbf{4}$ and $(+4) + (+1) = +\mathbf{5}$
So: $+\mathbf{5a}$ becomes $+4a + a$ to get 4 terms which can be **GROUPED**

Example 2: $b^2 - \mathbf{7b} + 6$

$$= b^2 - \mathbf{6b} - \boldsymbol{b} + 6$$
$$= (b^2 - 6b) + (-b + 6)$$
$$= (b^2 - 6b) - (\boldsymbol{b} - \mathbf{6})$$
$$= b(b - 6) - (b - 6)$$
$$= (b - 6)(b - 1)$$

Choose the **factors** of $+$ **6** such that if we add them, we get $-\mathbf{7}$, thus (-6) and (-1): $(-6)(-1) = +\mathbf{6}$ and $(-6) + (-1) = -\mathbf{7}$
So: $-\mathbf{7b}$ becomes $-6b - b$ to get 4 terms which can be **GROUPED**

Example 3: $x^2 + \mathbf{2x} - 15$

$$= x^2 + \mathbf{5x} - \mathbf{3x} - 15$$
$$= (x^2 + 5x) - (\mathbf{3x} + \mathbf{15})$$
$$= x(x + 5) - 3(x + 5)$$
$$= (x + 5)(x - 3)$$

Choose the **factors** of $-$ **15** such that if we add them, we get $+\mathbf{2}$, thus $(+5)$ and (-3): $(+5)(-3) = -\mathbf{15}$ and $(+5) + (-3) = +\mathbf{2}$
So: $+\mathbf{2x}$ becomes $+5x - 3x$ to get 4 terms which can be **GROUPED**

Example 4: $m^2 - m - 12$

$$= m^2 - \mathbf{4m} + \mathbf{3m} - 12$$
$$= (m^2 - 4m) + (3m - 12)$$
$$= m(m - 4) + 3(m - 4)$$
$$= (m - 4)(m + 3)$$

Choose the **factors** of $-$ **12** such that if we add them, we get $-\mathbf{1}$, thus (-4) and $(+3)$: $(-4)(+3) = -\mathbf{12}$ and $(-4) + (+3) = -\mathbf{1}$
So: $-\boldsymbol{m}$ becomes $-m + 3m$ to get 4 terms which can be **GROUPED**

Method 2:

If we look at the product: $(x + a)(x + b)$

we get: $x^2 + bx + ax + ab$

$= x^2 + (a + b)x + ab$

Factorise

trinomial

Since Factorisation is the opposite operation of products, to Factorise $\boldsymbol{x^2 + (a + b)x + ab}$ we note the following:

- we will have 2 brackets
- the 1st terms in the brackets (x and x) are factors of the 1st term (x^2) of the quadratic
- the 2nd terms in the brackets
  - are **factors of** the **last term** (ab) of the quadratic such that:
  - if we **add them**, we get the **midterm** of the quadratic

Hence: to factorise $a^2 + \mathbf{5a} + 4$

Factors of a^2 are: $\boldsymbol{a}$ and $\boldsymbol{a}$, **so our brackets are** $(\boldsymbol{a} + \square)(\boldsymbol{a} + \Delta)$

To get $\square$ and Δ we look at Factors of $+$ **4**, which are:

- $(\pm 2) \times (\pm 2)$ and if we add then we get $+4$ or -4
- $(\pm 4) \times (\pm 1)$ and if we add then we get $+5$ or -5
- **We looking for factors that add up to $+5$, hence $\square = +4$ and $\Delta = +1$**

$\therefore a^2 + \mathbf{5a} + 4 = (a + 4)(a + 1)$

Example 2: $b^2 - \mathbf{7b} + 6$

Factors of b^2 are: $\boldsymbol{b}$ and $\boldsymbol{b}$, **so our brackets are** $(\boldsymbol{b} + \square)(\boldsymbol{b} + \Delta)$

To get $\square$ and Δ we look at Factors of $+$ **6**, which are:

- $(\pm 2) \times (\pm 3)$ and if we add then we get $+5$ or -5
- $(\pm 6) \times (\pm 1)$ and if we add then we get $+7$ or -7
- **We looking for factors that add up to -7, hence $\square = -6$ and $\Delta = -1$**

$\therefore b^2 - \mathbf{7b} + 6 = (b - 6)(b - 1)$

Example 3: $x^2 + \mathbf{2x} - 15$

$= (x + 5)(x - 3)$

$(x \quad + 5)$
$(x \quad - 3)$
$x^2 + 2x - 15$

Example 4: $m^2 - \boldsymbol{m} - 12 = (m - 4)(m + 3)$

since $(-4)(+3) = -12$ and $(-4) + (+3) = -1$

NOTES: To factorise $x^2 + bx + c$

- 2 brackets
- 1st terms in brackets is x and x
- 2nd terms are factors of last term, c, such that, if they are added, it gives the desired middle term.
- If the sign of c is $(+)$, we **add** the factors to get the middle term; both will be $(+)$ if middle term is *positive*, or $(-)$ if middle term is *negative*
- If the sign of c is $(-)$, we "**subtract**" the factors to get the middle term; one factor will be $(+)$ and the other $(-)$
  - if middle term is *positive*, the bigger factor will be $(+)$
  - if middle term is *negative*, the bigger factor will be $(-)$

Example 5: **Factorise:** $x^2 - \mathbf{8}x + 12$

Solution:

$= (x-6)(x-2)$

$x \quad -6$
$x \quad -2$
$-6x - 2x = \mathbf{-8x}$

Example 6: **Factorise:** $3x^2 - 39x - 90$
Solution:

$= 3(x^2 - 13x - 30)$
$= 3(x-15)(x+2)$

$x \quad -15$
$x \quad +2$
$-15x + 2x = \mathbf{-13x}$

NOTE: Although $(-10) \times (+3) = -30$, if we add, we get -7

4. Factorisation: Trinomials - the Perfect square

As you've notice in the previous exercise, nr 6:
$x^2 - 8x + 16 = (x-4)(x-4) = (x-4)^2$ - the perfect square form of the trinomial – 1st term and last term are squares and the
midterm $= \pm \mathbf{2} \times \sqrt{(\textbf{1st term}) \times (\textbf{3rd term})}$

Example 1: **Factorise:** $x^2 + \mathbf{2x} + 1$
$= (x+1)^2$ ⇒Perfect square

$2x = 2 \times \sqrt{x^2 \times 1}$

Example 2: **Factorise:** $3x^2 - 30x + 75$
$= 3(x^2 - 10x + 25) = 3(x-5)^2$

Exercise 1
Factorise the following
(Use any method)

1. $x^2 + 10x + 9$
2. $x^2 - 10x + 16$
3. $a^2 - 4a - 12$
4. $m^2 + m - 30$
5. $6a^2 - 36a + 48$
6. $x^2 - 8x + 16$
7. $x^2 + 5xy - 36y^2$
8. $x - 6x^{\frac{1}{2}}y + 8y^2$
9. $5x^2 + 28x + 15$

Answers: Exercise 1

1. $(x+9)(x+1)$
2. $(x-8)(x-2)$
3. $(a-6)(a+2)$
4. $(m+6)(m-5)$
5. $6(a-4)(a-2)$
6. $(x-4)^2$
7. $(x+9y)(x-4y)$
8. $(x^{\frac{1}{2}} - 2y)(x^{\frac{1}{2}} - 4y)$
9. $(5x+3)(x+5)$

Exercise 2
Factorise the following completely
(Use any method)

1. $x^2 + 10x + 25$
2. $x^2 - 6x + 9$
3. $a^2 - 4a + 4$
4. $3m^2 + 36m + 108$
5. $(a^2 - 2ab + b^2) - w^2$
6. $4x^2 - 12xr + 9r^2 - 1$

Answers: Exercise 2

1. $(x+5)^2$
2. $(x-3)^2$
3. $(a-2)^2$
4. $3(m+6)^2$
5. $(a-b-w)(a-b+w)$
6. $(2x-3r+1)(2x-3r-1)$

Lesson 3 + 4:	**Factorisation:** The trinomial of the form $ax^2 + bx + c$

Example 1: Factorise: $4x^2 + 11x + 6$

Method 1: Multiply **4** with **6** = **+24**

$$4x^2 + 11x + 6$$
$$= 4x^2 + \mathbf{8x} + \mathbf{3x} + 6$$
$$= (4x^2 + 8x) + (3x + 6)$$
$$= 4x(x+2) + 3(x+2)$$
$$= (x+2)(4x+3)$$

Choose the **factors** of + **24** such that if we add them, we get $\mathbf{+11}$, thus $(+8)$ and $(+3)$: $(+8)(+3) = \mathbf{+24}$ and $(+8) + (+3) = \mathbf{+11}$

So: $\mathbf{+11x}$ becomes $+8x + 3x$ to get 4 terms which can be **GROUPED**

Method 2:

Since Factorisation is the opposite operation of products, to Factorise a quadratic trinomial:

- we will have 2 brackets
- * the 1st terms in the brackets are factors of the 1st term of the quadratic
- the 2nd terms in the brackets
  - are **factors of** the **last term** of the quadratic such that:
  - if we **cross-multiply** them with * respectively and **add them**, we should get the **middle term** of the quadratic

Hence: to factorise $4x^2 + \mathbf{11x} + 6$

Factors of $4x^2$ are: $2x$ and $2x$ or $4x$ and x

Factors of + 6 are: ± 6 and ± 1 or ± 2 and ± 3 [we only need to look at the (+) values since last term is (+) and the middle term is (+)]

Factors of 1st term		Factors of Last term			
$2x$	$4x$	+6	+1	+3	+2
$2x$	x	+1	+6	+2	+3

If we look at the 1st sets of factors: cross-multiply and add

$2x \quad +6 \Rightarrow +12x$

$2x \quad +1 \Rightarrow +2x$

$+14x$ not the middle term

Now look at

$4x \quad +3 \Rightarrow +3x$

$x \quad +2 \Rightarrow +8x$

$+11x$ which is the middle term

You don't have to show this process, but it is important

$\therefore 4x^2 + \mathbf{11x} + 6$

$= \mathbf{(4x+3)(x+2)}$

Example 2: $20x^2 + 24xy - 9y^2$

Method 1: $\mathbf{20x^2} + 24xy - \mathbf{9y^2}$

Signs must be different

$20 \times -9 = -180$

$\Rightarrow$ factors of -180 that adds up to $+24$: $+30$ and -6

$\therefore 20x^2 + 24xy - 9y^2$

$= 20x^2 + \mathbf{30xy} - \mathbf{6xy} - 9y^2$

$= (20x^2 + 30xy) + (-6xy - 9y^2)$

$= 10x(2x+3y) - 3y(2x+3y)$

$= (2x+3y)(10x-3y)$

Method 2: $\mathbf{20x^2} + 24xy - \mathbf{9y^2}$

Factors of $\mathbf{20x^2}$: $20x$ and x; $10x$ and $2x$; $4x$ and $5x$

Factors of $\mathbf{-9y^2}$: $+9y$ and $-y$; $-9y$ and $+y$; $+3y$ and $-3y$

Look for the arrangement that will give the desired middle term – **trial and error**

Take: $10x \quad -9y \Rightarrow -18xy$

$2x \quad +1y \Rightarrow +20xy$

$+2xy$

$10x$ and $2x$ are factors of $20x^2$;

$-9y$ and $+1y$ are factors of $-9y^2$

OR

$10x \quad +9y \Rightarrow +18xy$

$2x \quad -1y \Rightarrow -20xy$

$2xy$

Not the desired midterm: $\mathbf{+24xy}$; try other arrangements

Try: $10x \quad -3y \Rightarrow -6xy$

$2x \quad +3y \Rightarrow +30xy$

$\mathbf{+24xy}$ **YES!**

$= 20x^2 + 24xy - 9y^2$

$= (10x - 3y)(2x + 3y)$

The horizontal arrangements are the terms in the brackets!

Though practice it will become easier to get the factors

CAN YOU?

Exercise:

Factorise fully: Divide exercise between 2 days

1. $3a^2 + 10a + 3$
2. $5x^2 + 11x + 2$
3. $5a^2 - 23a + 12$
4. $8a^2 - 18a + 7$
5. $6x^2 + x - 1$
6. $10y^2 + 11y - 6$
7. $10p^2 + p - 9$
8. $21x^2 - 2x - 8$
9. $4x^2 - 28x - 15$
10. $9x^2 + 6xy + y^2$
11. $6s^2 - 11st + 3t^2$
12. $-2x^2 - 3xy - y^2$
13. $2x^2 - 20xy + 50y^2$
14. $10p^3 + 2p^2y - 8py^2$
15. $20a^2 - ab - 12b^2$
16. $12a^2 - 15ab - 18b^2$
17. $2x^2 - 1 - \frac{1}{x^2}$
18. $8f^2 + 3gf + \frac{1}{4}g^2$
19. $3x^2(x+2) - 16x(x+2) + 5(x+2)$
20. $15(a-b)^2 - 7(a-b) - 4$

Answers:

1. $(3a+1)(a+3)$
2. $(5x+1)(x+2)$
3. $(5a-3)(a-4)$
4. $(2a-1)(4a-7)$
5. $(3x-1)(2x+1)$
6. $(2y+3)(5y-2)$
7. $(10p-9)(p+1)$
8. $(7x+4)(3x-2)$
9. $(2x+1)(2x-15)$
10. $(3x+y)^2$
11. $(2s-3t)(3s-t)$
12. $-(2x+y)(x+y)$
13. $2(x-5y)^2$
14. $2p(5p-4y)(p+y)$
15. $(4a-3b)(5a+4b)$
16. $3(4a+3b)(a-2b)$
17. $\left(2x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$
18. $\left(2f+\frac{1}{2}g\right)\left(4f+\frac{1}{2}g\right)$
19. $(x+2)(3x-1)(x-5)$
20. $(5a-5b-4)(3a-3b+1)$

Lesson 5:	Factorisation: **Sum and Difference of Cubes**

Remember factorisation is the reverse of finding the product – changing a sum expression (polynomial) to a product expression (monomial)

Factorising the sum and difference of two cubes:

Cube: number raised to the power 3, e.g. x^3 (x cubed or x to the power 3) OR exponents divisible by 3

$\boldsymbol{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$ **: sum of 2 cubes**

$\boldsymbol{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$ **: difference of 2 cubes**

Method:

- 2 brackets
- 1st bracket: Take the $\sqrt[3]{\ }$ of each term in sum/ diff of cubes expression
- Sign depends on sum $(+)$ or difference $(-)$
- 2nd bracket:
    - square 1st term (from 1st bracket)
    - 1st term × 2nd term (**change sign**)
    - square 2nd term.

Example 1: Factorise fully: $a^3 + 27$

Solution:

$$a^3 + 27$$
$$= (a+3)(a^2 - 3a + 9)$$

Example 2: Factorise fully: $8x^3 - 216$

Solution:

$$8x^3 - 216$$
$$= (2x-6)(4x^2 + 12x + 36)$$

Example 3: Factorise fully: $(z-y)^3 + m^3$

Solution:

$$(z-y)^3 + m^3$$
$$= [(z-y)+m][(z-y)^2 - m(z-y) + m^2)]$$

Example 4: Factorise fully: $x^6 + y^6$

Solution:

$$x^6 + y^6 = (x^3 + y^3)(x^3 - y^3)$$
$$= (x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$$

- To check whether your factors are correct, you can always do the multiplication as rough work.

CAN YOU?

Factorise fully:

Exercise:

1. $125x^3 + y^3$
2. $216m^3 - b^3$
3. $x^3 - 125y^3$
4. $-216x^3 - y^3$
5. $x^3 + \dfrac{8}{x^3}$
6. $2x^4 - 128x$

Answers:

1. $(5x+y)(25x^2 - 5xy + y^2)$
2. $(6m-b)(36m^2 + 6mb + b^2)$
3. $(x-5y)(x^2 + 5xy + 25y^2)$
4. $-(6+y)(36 - 6y + y^2)$
5. $\left(x + \frac{2}{x}\right)\left(x^2 - 2 + \frac{4}{x^2}\right)$
6. $2x(x-4)(x^2 + 4x + 16)$

Gr 10: **Mixed Exercise (Test your knowledge)**

Factorise completely:

1. $12a^2b - 8ab + 4ab^2$
2. $4(a^2 - b^2) - b(b^2 - a^2)$
3. $4x^3 - 14x^2 + 12x$
4. $16w^2 - 14w + 3$
5. $4p^2 + 12pq + 9q^2$
6. $9x^2 - 18xw^2 + 8w^4$
7. $(25g^2 - 10g + 1) - 4h^2$
8. $8x^3 - 125y^3$
9. $3p^2 - 8pq - 35q^2$
10. $-3t^2 + 8ts + 16s^2$

Answers:

1. $4ab(3a - 2 + b)$
2. $(a + b)(a - b)(4 + b)$
3. $2x(x - 2)(2x - 3)$
4. $(8w - 3)(2w - 1)$
5. $(2p + 3q)^2$
6. $(3x - 2w^2)(3x - 4w^2)$
7. $(5g - 1 - 2h)(5g - 1 + 2h)$
8. $(2x - 5y)(4x^2 + 10xy + 25y^2)$
9. $(3p + 7q)(p - 5q)$
10. $-(3t + 4s)(t - 4s)$

ACTIVITIES	*Consider other exercises from your Mathematics Textbook*
CONSOLIDATION	**FACTORISATION**

Please do have a look at the Youtube videos for consolidation and other options of factorising quadratic trinomials

a) **<u>Common Factor</u> (CF):**

$ab + ac = a(b + c)$ OR $a(b + c) + d(b + c) = (b + c)(a + d)$

b) **<u>Factorising Two Terms</u>**

- Difference of 2 squares (DOTS):
  $\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$
- Sum of two cubes:
  $\boldsymbol{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$
- Difference of two cubes:
  $\boldsymbol{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$

c) **Terms remain the same (Cannot be factorised further)**

$(\boldsymbol{x^2 + y^2})$ both terms positive not DOTS

$(\boldsymbol{x + y})$ simplest form of expression

	d) <u>**Factorising Three terms**</u> • Perfect Square • Ordinary quadratic Trinomials **e)** <u>**Factorising Four terms i.e. by Grouping**</u>: Group terms in pairs, so that each pair has a **common factor** and preferably with a + sign between the pairs o Take out CF in each pair (considering sign changes) and continue as with CF as an expression o Special cases which involve either i) Perfect Square, folllowed by Difference of Two Squares ii) Sum/Difference of cubes, followed by Difference of two squares
VALUES	*Dear learner. Mathematics is a PRACTICE subject. That is why you will get Homework every day. Work daily at your Mathematics. PRACTICE makes PERFECT.*