Western Cape Government
Education

Directorate: Curriculum FET

SUBJECT and GRADE	Mathematics Grade 11	
TERM 1	Week 1	
TOPIC	Exponents and Surds	
AIMS OF LESSON	• Defining a rational exponent. • Simplifying expression • Solving exponential equations. • Defining surds • Identifying complex, simple and mixed surds • Simplifying surds • Solving equations containing surds.	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to EXPONENTS AND SURDS chapter in your textbook.	Where you see this icon in the lesson you can click on it to see a video on concepts and calculations of EXPONENTS AND SURDS.

INTRODUCTION

Dear learner in this chapter we review the laws of exponents and exponential equations. When we've covered that, we will have a look at rational exponents and surds. You will also learn how to solve exponential equations, simplify surds and solve equations containing surds.

Concepts and skills

Exponents: The exponent of a number tells us how many times to multiply the number (the base) by itself.

$\boxed{a^n}$ → **Exponent**, **Power**, **Base**

$64 = 2^6$ → **the exponent**, **the base**, **the power**

$a^n = a \times a \times a \times \ldots\ldots until\ there\ are\ n\ factors\ of\ a, where\ a\ \in R\ and\ n \in R$

$a^2 = a \times a$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

A. **LAWS of EXPONENTS**

These laws assume that $\boldsymbol{a}$ and $\boldsymbol{b}$ are positive real numbers

https://youtu.be/iTt3TVjsjuY

Laws of exponents only apply to multiplication, division, brackets and roots **and not to** addition and subtraction

No	Expanded Notation	Exponential Notation	Exponential laws in operation
1	$2 \times 2 \times 2 \times 2 = 16$	$2 \times 2 \times 2 \times 2 = 2^4$	When we multiply the same bases we ADD the exponents $\boldsymbol{a^n \times a^m = a^{n+m}}$
2	$\frac{64}{16} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 4$	$\frac{2^6}{2^4} = 2^2$	When we divide the same bases we subtract the exponents (top minus bottom) $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$
3	$4 \times 4 \times 4 = 64$	$\boldsymbol{(2^2)^3 = 2^6}$	When we have the exponents outside the bracket we distribute them into the bracket (exponent on the outside is multiplied by the exponent(s) on the inside) $\boldsymbol{(a^m)^n = a^{mn}}$
4	$2 \times 2 \times 3 \times 3 = 36$	$2^2 \times 3^2 = 6^2$	When we have non-identical bases, but identical exponents, we keep the exponents and multiply the bases (this same rule will also apply for division) $\boldsymbol{a^n b^n = (ab)^n}$
5	$\left(\frac{4}{2}\right)\left(\frac{4}{2}\right)\left(\frac{4}{2}\right) = \frac{64}{8} = 8$	$\left(\frac{4}{2}\right)^3 = \frac{4^{1\times3}}{2^{1\times3}} = \frac{4^3}{2^3}$	When a fraction is raised to a power, both the numerator and denominator are raised to the power $\boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$

Exponential definition:

$x^{-n} = \frac{1}{x^n}$	This definition allows us to move numbers or variables from the top to the bottom of a fraction, or bottom to top	$\frac{3}{x^{-2}} = 3x^2$ or $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
$x^0 = 1, \quad x \neq 0$	Any base (except zero) to the power of zero is equal to 1	$10^0 = 1$; $(2ab^2)^0 = 1$ NB: $-10^0 = -1$ but $(-10)^0 = 1$

Study and work through each concept above as to ensure that you understand the laws and exponential definitions.
We will now do some Gr 10 revision to recap our content knowledge and prepare us for new added content of Exponents and Surds

Worked Example 1

Let us work through a few examples of how to apply these laws

https://youtu.be/Hn_rbQ2zSAU

Simplify without using a calculator

1. $y^3 \times y$ **2.** $9^6 \div 9^4$ **3.** $(y\,x^2)^7$ **4.** $\left(\frac{2x^3}{8y^{-4}}\right)^{-3}$ **5.** $(-3a^2)^2(2a)^3$

Solutions [Can you identify all the exponent laws and definitions?]

1. $y^3 \times y$

$= y^3 \times y^1$

$= y^4$

2. $9^6 \div 9^4$

$= 9^{(6-4)}$

$= 9^2$

$= 81$

3. $(y\,x^2)^7$

$= y^{(1\times7)}x^{(2\times7)}$

$= y^7x^{14}$

4. $\left(\frac{2x^3}{8y^{-4}}\right)^{-3}$

$= \left(\frac{8y^{-4}}{2x^3}\right)^3$

$= \left(\frac{4}{x^3y^4}\right)^3$

$= \frac{4^3}{x^{3\times3}y^{4\times3}} = \frac{64}{x^9y^{12}}$

5. $(-3a^2)^2(2a)^3$

$= 9a^4.8a^3$

$= 72a^7$

Can you?

Apply the laws to simplify the following without the use of a calculator

1. $y^{2n}.y^5$ 2. $a^{3n} \div a^n$ 3. $(x^2)^4$ 4. $\left(\frac{2x^2}{3y^3}\right)^3$ 5. $(-3y^3)^2$

Answers:

1. y^{2n+5} 2. a^{2n} 3. x^8 4. $\frac{8x^6}{27y^9}$ 5. $9y^6$

Rational numbers include all numbers that can be written as fractions in the form $\frac{a}{b}$ *where* $a, b \in Z$ *but* $b \neq 0$

B. Simplifying expressions involving rational exponents

When compound numbers are raised to a rational exponent, it can be simplified by using prime factors and the rule for raising a power to another power e.g. $\mathbf{32^{\frac{3}{5}} = (2^5)^{\frac{3}{5}} = 2^3 = 8}$

Consider the following $\sqrt[n]{a} = a^{\frac{1}{n}}$ if we raise both sides to the power m we have

$$(\sqrt[n]{a})^m = (a^{\frac{1}{n}})^m$$

Therefore $\qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$

Work through all the calculations in the worked examples and identify laws and definitions

Worked Examples 2

Simplify the following expressions, without a calculator and leave your answers with positive exponents.

1. $(8)^{\frac{1}{3}}$ $\quad$ 2. $(\frac{1}{4})^{-2}$ $\quad$ 3. $(8a^6b^{12})^{\frac{1}{3}}$ $\quad$ 4. $\sqrt[5]{32^2}$ $\quad$ 5. $(\frac{-2x^{-2}}{(-2x)^{-2}})^{-\frac{1}{3}}$

Solutions:

1. $(8)^{\frac{1}{3}}$

$= (2^3)^{\frac{1}{3}}$

$= 2$

2. $(\frac{1}{4})^{-2}$

$= 4^2$

$= 16$

3. $(8a^6b^{12})^{\frac{1}{3}}$

$= (2^3a^6b^{12})^{\frac{1}{3}}$

$= 2a^2b^4$

4. $\sqrt[5]{32^2}$

$= \sqrt[5]{(2^5)^2}$

$= (2^5)^{\frac{2}{5}}$

$= 2^2 = 4$

5. $(\frac{-2x^{-2}}{(-2x)^{-2}})^{-\frac{1}{3}}$

$= (\frac{-2x^{-2}}{(-2)^{-2}\,x^{-2}})^{-\frac{1}{3}}$

$= (-2x^{-2}.(-2)^2x^2)^{\frac{-1}{3}}$

$= (-2^3)^{\frac{-1}{3}}$

$= -2^{-1} = -\frac{1}{2}$

CAN YOU simplify the following expressions, without the use of a calculator? Your answers should have positive exponents.

1. $\left(\frac{1}{2}\right)^{-3}$ $\quad$ **2.** $3^{-1}.2^0$ $\quad$ **3.** $\left(\frac{\sqrt{x}}{x^{\frac{-3}{2}}}\right)^{\frac{1}{2}}$ $\quad$ **4.** $\left(3\,\frac{3}{8}\right)^{\frac{-2}{3}}$ $\quad$ **5.** $\sqrt[3]{\frac{27a^3b^6}{64c^9}}$

Answers:

1. 8 $\quad$ **2.** $\frac{1}{3}$ $\quad$ **3.** x $\quad$ **4.** $\frac{4}{9}$ $\quad$ **5.** $\frac{3ab^2}{4c^3}$

C. Simplifying expressions using factors

The questions that you would need to answer can be divided into one of the two types

Type 1: How to simplify expressions consisting of one term

When the expression consists of one term (only multiplication and division) factorise into prime factors and apply the exponential laws.

Follow and understand the following steps

1. Rewrite all composite numbers as a product of their prime factors.
2. Use exponential laws to raise all powers appropriately.
3. Use the rule for division to move all bases into the numerator
4. Then simplify

$$\frac{a \rightarrow \ Numerator}{b \ \rightarrow denominator}$$

Worked example 3

Simplify:

$$\frac{25^n.36^{n+1}}{81.\ 30^{2n}}$$

Factorise into prime numbers

$$= \frac{(5^2)^n.(2^2.3^2)^{n+1}}{3^4.(2.3.5)^{2n}}$$

Apply exponent laws

$$= \frac{5^{2n}.2^{2n+2}\ .3^{\ 2n+2}}{3^4.2^{2n}3^{2n}.5^{2n}}$$

*

$$= \mathbf{2^{2n+2-2n}.3^{2n+2-4-2n}}$$

$$= \mathbf{2^2.3^{-2}}$$

$$= \mathbf{\frac{4}{9}}$$

Remember $3^{-2} = \frac{1}{3^2}$

Type 2: How to simplify expressions consisting of more than one term

When an expression consists of more than one term you have to factorise the expression before you can simplify (terms are separated by + or – signs)

Follow and understand the following steps

1. Separate and factorise
2. Simplify further using your knowledge of common fractions

Worked example 4

Simplify: $3^{x+1} + 3^x$ [the common factor in each term is 3^x]

$= 3^x(3+1)$ Divide by the common factor

[$3^{x+1} \div 3^x = 3$ and $3^x \div 3^x = 1$]

$= 3^x.4$

Worked example 5: Simplify:

$$\frac{2^{x+3} - 2.2^x}{2^x}$$

$$= \frac{2^x.2^3 - 2.2^x}{2^x}$$

$$= \frac{2^x(8-2)}{2^x}$$

$$= 6$$

Can you ?

Simplify the following expressions

1. $\frac{8^{n+3}.32^{-n-1}.6^{2n}}{9^n}$
2. $\frac{2.3^{x-1} - 3^x}{3^x - 3^{x-1}}$

3. Write down the values of a, b and c

$64^a = 8$ $\quad$ $64^b = 4$ $\quad$ $64^c = 2$

Answers

1. 16 $\quad$ 2. $-\frac{1}{2}$

3. $a = \frac{1}{2}$ $\quad$ $b = \frac{1}{3}$ $\quad$ $c = \frac{1}{6}$

D. Exponential Equations

There are two types of exponential equations:

- **Type 1:** Equations with the exponent as the unknown
- **Type 2:** Equations with the base as the unknown

Type 1: Variables are in the exponent

This type of exponential equation with the variable in the exponent can also be given or simplified to one of the following structures:

I. Equations with one term on either side of the equation e.g. $a^x = a^b$, then $x = b$, for $a \neq 0$

II. Equations with more than one term with the variable, x, e.g. $3^x = 3^{x-2} + 24$

III. Equations where one of the terms has an exponent of, "$2x$" and another term an exponent of, "x", e.g $5^{2x} - 4.5^x - 5 = 0$, this results in a quadratic equation.

IV. Equation where one of the terms has an exponent, "$-x$" and another term has an exponent of "x", e.g $2^{x+1} + 2^3.2^{-x} = 17$, this also results in a quadratic equation.

I. Equations with one term on either side of the equation

This type of equation is using the basic premise that if $\boldsymbol{a^x = a^b}$ **, then** $\boldsymbol{x = b}$ **, for** $\boldsymbol{a \neq 0}$

For example, consider the equation $3^x = 9$. The equation can be solved as follows:

$3^x = 9$

$\therefore 3^x = 3^2$ write 9 to the base of 3

$\therefore x = 2$ Equate the exponents

The intention is therefore to express both sides of the equation with the same base so that we can equate the exponents.

Worked examples 6

If $x \in R$, solve for x in the following equations:

a) $4^{x-1} = 8^{-1}$ b) $(5^{x-2})^x = 125$

Study and work through this worked examples step by step until you understand each calculation.

Solutions:

a) $4^{x-1} = 8^{-1}$

$(2^2)^{x-1} = (2^3)^{-1}$ express each term as a product of its prime factors

$2^{2x-2} = 2^{-3}$

$2x - 2 = -3$ equate the exponents

$2x = -1$

$\therefore x = -\frac{1}{2}$

b) $(5^{x-2})^x = 125$

$5^{x^2-2x} = 5^3$ express each term as a product of its prime factors

$\therefore x^2 - 2x = 3$ equate the exponents

$x^2 - 2x - 3 = 0$ factorise to solve the trinomial

$(x-3)(x+1) = 0$

$\therefore x = 3$ or $x = -1$

CAN YOU solve for x in the following equations?

1. $9^{x+1} = 27^x$
2. $4 \,.\, 3^{7x} = 9 \,.\, 2^{7x}$

Answers:

1. $x = 2$
2. $x = \frac{2}{7}$

Continuation of

D. Exponential Equations

Type 1: Equations with the exponent as the unknown

II. Equations with more than one term with the same variable, e.g $3^x = 3^{x-2} + 24$

Worked examples 7

Solve for x:

$$3^x = 3^{x-2} + 24$$

Make sure the terms containing an x is taken to one side.

Solution:

$3^x - 3^{x-2} = 24$

$3^x(1 - 3^{-2}) = 24$ Factorise using common factors

$3^x\left(\frac{8}{9}\right) = 24$ Simplify

$3^x = 24 \times \frac{9}{8}$

$3^x = 27$ Express each term as a product of its prime factors

$3^x = 3^3$ equate the exponents once the bases are equal

$\therefore x = 3$

III. Equations where one of the terms has an exponent of, "$2x$" and another term an exponent of, "x", e.g $5^{2x} - 4.5^x - 5 = 0$, this results in a quadratic equation.

Worked examples 8

Solve for x: $5^{2x} - 4.5^x - 5 = 0$

notice, one term with exponent "$2x$" and another term with an exponent of, "x"

Solution:

Let $5^x = k$

$k^2 - 4k - 5 = 0$

$(k-5)(k+1) = 0$

$k = 5$ or $k = -1$

$5^x = 5^1$ or $5^x = -1$

$\therefore x = 1$

Factorise the quadratic equation
Solve for k
Then solve for exponent "x"

Since there is no value of x that will give us a negative value in the second solution, i.e $5^x = -1$, this equation has no solution.
Therefore, $x = 1$ is the only valid solution

IV. Equation where one of the terms has an exponent, "$-x$" and another term has an exponent of "x", e.g $2^{x+1} + 2^3.2^{-x} = 17$, this also results in a quadratic equation.

Worked examples 9

Solve for x:

$2^{x+1} + 2^3.2^{-x} = 17$

Notice, one term with exponent "$-x$" and another term has an exponent of "x"

Solution:

$2^x.2^1 + 2^3.\left(\frac{1}{2^x}\right) = 17$ write 2^{-x} as $\frac{1}{2^x}$

$k.2^1 + 2^3\left(\frac{1}{k}\right) = 17$ Let $2^x = k$ and multiply by k throughout

$2k^2 + 8 = 17k$

$2k^2 - 17k + 8 = 0$ factorise the quadratic equation

$(2k-1)(k-8) = 0$

$k = \frac{1}{2}$ or $k = 8$ Solve for k

$2^x = 2^{-1}$ or $2^x = 2^3$ Solve for x

$\therefore x = -1$ or $x = 3$

CAN YOU solve for x if $x \in R$

1. $4^{x+1} - 64 = 0$
2. $3.3^{2x} - 4.3^x = -1$

Answer:

1. $x = 2$ 2. $x = -1$ or 0

Continuation of

D. Exponential Equations

Type 2: Variable x, is in the base

If $x^{\frac{m}{n}} = c$, where c is any constant then:

- If m is odd, then there is only one solution.
- If m is even, then there are two solutions, one positive and one negative

If the unknown variable (say x) is a base, we raise both sides to the same power (the reciprocal of the power of x) in order to change the exponent of the variable to 1

Worked example 10

Solve for x if $x \in R$

a) $2x^{\frac{2}{3}} = 32$

b) $\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = (64)^{-\frac{2}{3}}$

Solution:

a) $2x^{\frac{2}{3}} = 32$ [first divide by 2, which is the coefficient of x]

$x^{\frac{2}{3}} = 16$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(16)^{\frac{3}{2}}$

$x = \pm(2^4)^{\frac{3}{2}}$ [express 16 as a product of its prime factors]

$x = \pm(2)^6$

$\therefore x = \pm 64$

b) $x^{-\frac{3}{2}} = 64$

$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = (64)^{-\frac{2}{3}}$ Please note that the reciprocal power must also be negative

$x = (2^6)^{-\frac{2}{3}}$

$x = 2^{-4}$

$\therefore x = \frac{1}{16}$

CAN YOU?

1. Solve for x if $3x^{\frac{5}{2}} = 96$
2. Solve for x if $3x^{-\frac{5}{3}} + 16 = 112$

Answers:

1. $x = 4$
2. $x = 2^{-3} = \frac{1}{8}$

E) Simplification of Surds

https://youtu.be/hcsHHWvNZWo

- Definition: a surd is the root of a whole number that produces an irrational number.
- Therefor a surd is the root of a number that cannot be determined exactly.
- An irrational number is a number that cannot be expressed as an integer or as a fraction that results in a finite number of digits (i.e. is a number that is non- recurring, non-terminating decimal)
- Examples would be $\sqrt{3}$, $\sqrt{7}$ $\sqrt[3]{5}$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- If $\sqrt[n]{a} = x$, then $x^n = a$

Laws of surds	**examples**	**Explanatory notes**
$\sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{x.y}$	• $\sqrt{5} \times \sqrt{3} = \sqrt{15}$ • $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$	When surds are multiplied then the numbers under the surd can be multiplied and then the surd is applied to the product. $\sqrt{20}$ is not in its simplest form because a perfect square is a factor of 20 i.e $20 = 5 \times 4$
$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	• $\frac{\sqrt{3}}{\sqrt{18}} = \sqrt{\frac{3}{18}} = \sqrt{\frac{1}{6}}$ • $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$	When surds are divided then the numbers under the surd is divided and the of the quotient is taken. apart and rooted individually
$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$	• $5\sqrt{6} - 2\sqrt{6} = 3\sqrt{6}$	If the root is of the same number or variable surds are treated the same as like terms. It can be added or subtracted, and the root remains the same.
$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$	• $\sqrt{\sqrt[3]{5^2}} = \sqrt[6]{5^2} = 5^{\frac{2}{6}} = 5^{\frac{1}{3}}$ • $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$	When taking the root of a root, it is the same as, taking the single root to the product of both roots.

Using the laws and definitions of surds

As with exponential problems, you will lose marks if you do not show your workings, including writing composite numbers as powers of prime numbers or as prime factors (only use your calculator to check your answers.)

Multiplication and Division of surds

Worked examples 11:

11.1 Simplify the following expressions without using a calculator. Show all your calculations.

a) $\left(\sqrt{4}\right)^2 =$ **b)** $\dfrac{\sqrt[3]{16}}{\sqrt[3]{2}} =$ **c)** $\dfrac{\sqrt{12}\times\sqrt{24}}{\sqrt{8}} =$

11.2 Determine, without the use of the calculator, which is greater $\sqrt{7}$ or $\sqrt[3]{15}$?

Work through each worked example and apply the multiplication and division laws of surds

$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{a.b}$ and $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Solutions:

11.1

a) $\left(\sqrt{4}\right)^2$

$= \sqrt{4}\times\sqrt{4}$

$= \sqrt{16}$

$= 4$

b) $\dfrac{\sqrt[3]{16}}{\sqrt[3]{2}}$

$= \sqrt[3]{\dfrac{16}{2}}$

$= \sqrt[3]{8}$

$= 2$

c) $\dfrac{\sqrt{12}\times\sqrt{24}}{\sqrt{8}}$

$= \dfrac{\sqrt{4}\times\sqrt{3}\times\sqrt{4}\times\sqrt{3}\times\sqrt{2}}{\sqrt{4}\times\sqrt{2}}$

$= \dfrac{2\sqrt{3}\times 2\sqrt{3}}{2}$

$= 2\sqrt{9}$

$= 2\times 3 = 6$

11.2

$\sqrt{7} = 7^{\frac{1}{2}} = 7^{\frac{3}{6}} = \sqrt[6]{7^3} = \sqrt[6]{343}$

$\sqrt[3]{15} = 15^{\frac{1}{3}} = 15^{\frac{2}{6}} = \sqrt[6]{15^2} = \sqrt[6]{225}$

$\sqrt[6]{343} > \sqrt[6]{225}$

$\therefore \sqrt{7} > \sqrt[6]{15}$

Express all as $\sqrt[6]{}$

CAN YOU

1. Simplify the following expressions?

a) $2\sqrt{3}.5\sqrt{3}$ **b)** $\left(3\sqrt{2}+3\right)\left(3\sqrt{2}-3\right)$

c) $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{32}}$ d) $\sqrt{21}.\sqrt{60}.\sqrt{35}$

2. Arrange in descending order $\sqrt[3]{5};\ \sqrt{3};\ \sqrt[6]{26}$

Answers:

1. a) 30 b) 9 c) 2 d) 210

2. $\sqrt{3};\ \sqrt[6]{26};\ \sqrt[3]{5}$

Addition and subtraction of surds

Worked example 12: Simplify each of the following

12.1 $4\sqrt{2}+\sqrt{3}-\sqrt{2}-2\sqrt{3}$ 12.2 $\frac{\sqrt{48}-\sqrt{3}}{\sqrt{27}}$ 12.3 $2\sqrt{8}-4\sqrt{32}+3\sqrt{50}$

Solutions:

12.1 $4\sqrt{2}+\sqrt{3}-\sqrt{2}-2\sqrt{3}$ [Remember only like surds can be added or subtracted … Think of it as, $[4x+y-x-2y=3x-y]$]

$= 3\sqrt{2}-\sqrt{3}$

12.2 $\frac{\sqrt{48}-\sqrt{3}}{\sqrt{27}}$

$= \frac{\sqrt{16\times 3}-\sqrt{3}}{\sqrt{9\times 3}}$ [write each number as a product of a square number and another number]

$= \frac{4\sqrt{3}-\sqrt{3}}{3\sqrt{3}}$ [use the surd laws to separate the square roots]

$= \frac{3\sqrt{3}}{3\sqrt{3}} = 1$

12.3 $2\sqrt{8}-4\sqrt{32}+3\sqrt{50}$

$= 2\times 2\sqrt{2}-4\times 4\sqrt{2}+3\times 5\sqrt{2}$ simplify surds using prime factors

$= 4\sqrt{2}-16\sqrt{2}+15\sqrt{2}$ Add and subtract like terms

$= 3\sqrt{2}$

Note to remember

$\sqrt{8}=\sqrt{4\times 2}=2\sqrt{2}$

$\sqrt{32}=\sqrt{16\times 2}=4\sqrt{2}$

$\sqrt{50}=\sqrt{25\times 2}=5\sqrt{2}$

CAN YOU simplify the following surds?

1. $\sqrt{7}+2\sqrt{7}-6\sqrt{7}$
2. $\frac{\sqrt{75}+\sqrt{48}}{\sqrt{12}}$
3. $\sqrt{50}(\sqrt{18}+\sqrt{32})$

Answers:

1. $-3\sqrt{7}$ **2.** $4\frac{1}{2}$ **3.** 70

F) Rationalising the denominator https://youtu.be/w3PGLLT5nr4

The principal of rationalising the denominator is to remove the surds(irrational numbers)in the denominator.

It is generally accepted that an answer is not completely simplified if there is a root/ surd in the denominator.

$\frac{a}{\sqrt{b}}=\frac{a}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}}=\frac{a\sqrt{b}}{b}$ To rationalize the denominator, the aim is to multiply by a version of 1 that will remove the surd from the denominator.

Remember $\sqrt{a}\times\sqrt{a}=a$

Worked Example 13: Rationalise the denominator for each of the following:

13.1 $\frac{5}{\sqrt{11}}$ 13.2 $\frac{3}{2+\sqrt{2}}$

Solution: 13.1 $\frac{5}{\sqrt{11}}$

$= \frac{5}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$

$= \frac{5\sqrt{11}}{11}$

13.2 $\frac{3}{2+\sqrt{2}}$

$= \frac{3}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{3(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$

$= \frac{6-3\sqrt{2}}{2}$

CAN YOU?

Rationalise the denominator of:

1. $\frac{6}{\sqrt{3}}$ 2. $\frac{5}{4-\sqrt{3}}$

Answers: 1. $2\sqrt{3}$ 2. $\frac{20+5\sqrt{3}}{13}$

F) Surd Equations

Solving equations involving surds:

- To solve this type of equation we use the basic premise that $\left(\sqrt{x}\right)^2 = x$
- To solve equations with surds, rearrange the equation so that the term containing the surd (or root sign: $\sqrt{\ }$) is on its own on one side of the equation, otherwise the middle term, when squaring, will again contain a surd.
- The validity of all possible solutions MUST be checked by substitution into the original equation.
- From gr 10 content we know that: if $x \geq 0$ then the expression $\sqrt{x}$ will be a real number.
- For example, $\sqrt{16}$; $\sqrt{5}$; $-\sqrt{6}$ are **real** numbers since the numbers under the square root signs are positive.
- However, if $x < 0$, then the expression $\sqrt{x}$ will be a **non-real** number.
- For example, $\sqrt{-1}$; $\sqrt{-4}$; $\sqrt{-16}$ are non- real numbers since the numbers under the square root signs are negative.
- If a particular solution does not satisfy the original equation, it is not a solution.

Work through the worked example until you are confident that you understand **and know** why and how to do each calculation.

Worked examples 14: Solve for x in $\sqrt{2x-4} + x = 6$

Solution

$\sqrt{2x-4} + x = 6$

$\boldsymbol{\sqrt{2x-4}} = 6 - x$ [Isolate the surd term]

$\left(\sqrt{2x-4}\right)^2 = (6-x)^2$ [square both sides of the equation]

$2x - 4 = 36 - 12x + x^2$

$0 = 36 + 4 - 12x - 2x + x^2$

$x^2 - 14x + 40 = 0$ [solve the equation]

$(x-10)(x-4) = 0$

$\therefore x = 10$ or $x = 4$

$x \neq 10$ and $x = 4$ is the only solution

Check the answers for validity

If $x = 10$

LHS $= \sqrt{2(10)-4} + 10$

$= \sqrt{16} + 10$

$= 14 \neq 6$ $\therefore LHS \neq RHS$

Check the answers for validity

If $x = 4$

LHS $= \sqrt{2(4)-4} + 4$

$= \sqrt{4} + 4$

$= 6$ $\therefore LHS = RHS$

Note: This follows from the above checking

CAN YOU?

1. Solve for x in $\sqrt{x+2}-x=0$
2. Solve for x in $\sqrt{3x+4}=2x+3$

Answers:

1. $x=2$ 2. $x=-1$ or $x=-\frac{5}{4}$

Consolidation

- Remember to revise the number systems as to ensure that you know all the different types of numbers.
- Exponents and Surds is part of Algebraic expressions which counts about 30% of the final Paper 1 examination
- A sound knowledge of exponents will assist you in Calculus in Grade 12.
- Surd equations need to be covered thoroughly – learners must test whether their solutions satisfy the original equation
- Practice by working out old question papers to get acquainted with the way question are asked in exams.

ACTIVITY	**Mind Action Series**	**Via Africa**	**Siyavula**	Classroom mathematics	Platinum
	Revision exercise pg 18	Summary and questions pg. 18 -19	End of chapter exercises Pg 16 and 17	Revision and consolidation Pg 22	Revision Pg 20 - 21