SUBJECT and GRADE	Mathematics Grade 11	
TERM 1	Week 2	
TOPIC	Quadratic Equations	
AIMS OF LESSON	• How to solve equations by using factors and using the quadratic formula. • How to solve quadratic equations that contains fractions • Solving quadratic equations using substitution (*k*- method)	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Equations and Inequalities chapter in your textbook.	Where you see this icon in the lesson you can click on it to see a video that would assist you in understanding the content that is being discussed

INTRODUCTION

Dear learner in grade 10 you have learned about equations and inequalities, covering content on:

- Linear equations, and that they are equations of the first degree
- How to solve linear equations
- How to use the Lowest Common Denominator (LCD) to solve linear equations that contains fractions.
- About quadratic equations, that are equations of the second degree
- How to solve quadratic equations
- How to solve simultaneous equations using the elimination method and the substitution method
- About linear inequalities, including how to represent them on a number line.

Revise this content from your grade 10 book. It will support you to better understand the grade 11 content.

In the following lessons we will build on this grade 10 content knowledge to cover grade 11 and 12 content knowledge

CONCEPTS AND SKILLS

Quadratic equations

Definition:

A quadratic equation is an equation in the form: $\boldsymbol{ax^2 + bx + c = 0}$ where *a, b* and *c* are constants and also $\boldsymbol{a \neq 0}$

This equation is in standard form: example $\boldsymbol{2x^2 + 3x - 4 = 0}$ where $\boldsymbol{a = 2,\ b = 3\ and\ c = -4}$

It is also known as a second-degree equation, because the largest exponent of the variable is 2. The solutions to the equation are also called the **roots** of the equation. The roots of the quadratic equation are the values of *x* that satisfy the equation, i.e. that will make the equation true.

A) Using factors to solve quadratic equations

https://video.tutonic.org/T11quadraticequations

Factorisation is used to simplify expressions and to solve quadratic equations.

The basis of solving by factors is **the principle of a Zero Product:**

If $a \times b = 0$, *then either* $a = 0$ *or* $b = 0$

> **Note** that there are three methods for solving quadratic equations:
> 1. Factorization
> 2. The quadratic formula and
> 3. Completing the square.
>
> **We will focus on the first two methods**

Factorisation commonly used when solving quadratic equations	
Common factors	$ax^2 + 2x = x(ax + 2)$
Difference of squares	$x^2 - 4x^2 = (a - 2b)(a + 2b)$
Trinomial factorisation	$a^2 + ab - 2b^2 = (a - b)(a + 2b)$ $a^2 - 3ab + 2b^2 = (a - b)(a - 2b)$ $a^2 + 3ab + 2b^2 = (a + b)(a + 2b)$ $a^2 - ab - 2b^2 = (a + b)(a - 2b)$
Grouping (common brackets)	$(a + b) - 3a(a + b) = (a + b)(1 - 3a)$
Difference of two cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

> Learners please revise and ensure that you remember and understand the different types of factorisation done in previous grades.

Let us work through a few examples:

Worked Example

1. Solve for x: $2x^2 - 3x = 0$

$2x^2 - 3x = 0$ [it is in standard form]

$x(2x - 3) = 0$ [factorise – common factor]

$x = 0$ or $2x - 3 = 0$ [zero product principle]

$\therefore 2x = 3$

$x = 0 \ \ or \ \ x = \frac{3}{2}$

Roots of equation

2. Solve for x : $(2x - 1)(x + 3) = 0$

$(2x - 1)(x + 3) = 0$ [both equations are already factorized and equal to zero… ready to be solved]

$2x - 1 = 0 \ or \ x = 3 = 0$

$2x = 1$ or $x = -3$

$x = \frac{1}{2}$ or $x = -3$

Check the roots $\frac{1}{2}$ and -3 of the equation, $(2x - 1)(x + 3) = 0$

For $x = \frac{1}{2}$: LHS = $\left(2(\frac{1}{2}) - 1\right)\left(\frac{1}{2} + 3\right) = 0 \times 3\frac{1}{2} = 0$ RHS and

For $x = -3$: LHS $(2(-3) - 1)((-3) + 3) = -7 \times 0 = 0$ RHS

$\therefore \frac{1}{2}$ *and* -3 satisfy the equation, as each of these values for x made RHS=LHS

3. Solve for a: $2a(a-1)+3(a-1)=0$

Solution:

$(a-1)(2a+3)=0$ [2 terms with a common factor of $(a-1)$]

$a-1=0$ *or* $2a+3=0$ [factorise by grouping]

$a=1$ or $2a=-3$

$a=1$ or $a=-\frac{3}{2}$

5. Solve for x: $2(x-2)(x+2)+6x=(x-1)^2$

Solution:

$2(x^2-4)+6x=x^2-2x+1$ [multiply out]

$2x^2-8+6x=x^2-2x+1$ [add like terms]

$x^2+8x-9=0$ [write in standard form]

$(x+9)(x-1)=0$

$x+9=0$ *or* $x-1=0$

$x=-9$ or $x=1$

4. Solve for x: $x^2-3x=10$

Solution:

$x^2-3x-10=0$ [write in standard form]

$(x-5)(x+2)=0$ [factorise]

$\therefore x-5=0$ *or* $x-2=0$ [zero product principle]

$x=5$ *or* $x=-2$

CAN YOU

Solve for *x* the following quadratic equations

1. $x^2-9=0$
2. $x(x+6)=0$
3. $x^2=5x+6$
4. $(3-x)(5-x)=3$

Answers:

1. $x=-3$ *or* $x=3$ 2. $x=0$ *or* $x=-6$ 3. $x=6$ *of* $x=-1$
4. $x=2$ *or* $x=6$

Quadratic equations involving fractions

We solve quadratic equations that contains fractions in the same way that we did our previous equations.
However, if one or more denominators contains variables, then there are restrictions on our solutions.
For example, if a denominator contains $(x+1)$, then this means that $x+1 \neq 0$, otherwise we would be dividing by 0.
Remember that dividing by 0 is undefined in mathematics.
Let us now do a few examples to show how to solve quadratic equations that include fractions

Worked examples:

6. Solve for x: $\frac{x-2}{x-1} = \frac{2x-1}{x+7}$

Solution:

LCD is $(x-1)(x+7)$ [find the LCD]

$\therefore \frac{(x-2)}{(x-1)}\frac{(x+7)}{(x+7)} = \frac{(2x-1)}{(x-1)}\frac{(x-1)}{(x+7)}$ $x \neq 1,\ x \neq -7$ [sort out restrictions]

$(x-2)(x+7) = (2x-1)(x-1)$

$x^2 + 5x - 14 = 2x^2 - 3x + 1$

$x^2 - 8x + 15 = 0$ [simplify to standard form]

$(x-3)(x-5) = 0$ [factorise]

$\therefore x = 3 \ \ or \ \ x = 5$ [both are valid solutions because $x \neq 1 \ \ or -7$]

7. Solve for x $\frac{x}{2x-4} - \frac{x}{x-2} = 1$

Solution:

$\frac{x}{2(x-2)} - \frac{x}{x-2} = 1$ [factorise denominator of fraction]

LCD is $2(x-2)$ [find the LCD]

$$2(x-2) \times \frac{x}{2(x-2)} - 2(x-2) \times \frac{x}{x-2} = 1 \times 2(x-2)$$

$$x - 2x = 2x - 4$$

$$-2x = 2x - 4$$

$$-3x = -4$$

$$x = \frac{4}{3}$$

CAN YOU?

Solve for x in the following equations

1. $\frac{3}{x-4} + \frac{x-3}{x} = 2$
2. $\frac{x+2}{x+1} - \frac{3}{x-2} = \frac{1}{x+1}$

Answers:

1. $x = 6 \ \ or \ x = -2$
2. $x = 5$

Solving quadratic equations using substitution (k-method)

We can simplify an equation by replacing one of the ***complex variables*** common factor terms with simpler variables. This will make a complicated equation much easier to work with. Important to remember to reverse the substitution at the end. This is called using the *k*- method.
Let us now do a few examples to see how to use the *k*- method.

Worked examples:

8. **Solve for *x*:** $\mathbf{2(x+5)^2+3(x+5)-2=0}$

Solution:

Let $(x+5)=k$ …we can use any variable as long as it does not appear in the original equation

$$2(k)^2+3(k)-2=0$$
$$(2k-1)(k+2)=0$$
$$2k-1=0 \quad or \quad k+2=0$$
$$k=\frac{1}{2} \quad \text{or} \quad k=-2$$

But $\boldsymbol{k}= \boldsymbol{x+5}$ $\quad \therefore x+5=\frac{1}{2} \quad or \;\; x+5=-2$

$$x=4\frac{1}{2} \quad \text{or} \quad x=-7$$

Worked example

9. **Solve for:** $\mathbf{3x^2+x-1+\frac{1}{3x^2+x-3}=0}$

Solution*:*

Let $3x^2+x=k$ $\quad \therefore k-1+\frac{1}{k-3}=0$

$$\frac{(k-1)(k-3)}{k-3}=0$$
$$(k-1)(k-3)+1=0$$
$$k^2-4k+4=0$$
$$(k-2)^2=0$$
$$k-2=0$$
$$\therefore 3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$
$$\therefore x=\frac{2}{3} \; or \; x=-1$$

CAN YOU

Solve for x in the following quadratic equations.

1. $(x^2+3)^2-2(x^2+3)-8=0$
2. $\frac{1}{x^2-x-1}=x^2-x-1$

Answers: 1. $x=-4 \; or \; 1 \; or -2 \; or -1$ $\qquad$ 2. $x=0 \; or \; 1 \; or \; 2 \; or -1$

Using the quadratic formula to solve quadratic equations

- The quadratic formula can be used to find the roots of any quadratic equation in the form, $\boldsymbol{ax^2 + bx + c = 0}$ **.**
- It is used to find roots of quadratic equations that cannot be factorized.
- The quadratic formula is:

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

- When you use the quadratic formula, always ensure that the equation is in the standard form $\boldsymbol{ax^2 + bx + c = 0}$
- Then substitute *a, b and c* mindfully.
- a is the coefficient of x^2, b is the coefficient of x and c is the constant term

Worked example:

11. Solve for x by using the quadratic formula

$$3x = -2x^2 - 5$$

Solution:

$2x^2 + 3x + 5 = 0$ [standard form]

$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ [quadratic formula]

$\boldsymbol{x = \frac{-3 \pm \sqrt{(3) - 4(2)(5)}}{2(2)}}$ [substitution]

$x = -3 \pm \frac{\sqrt{-31}}{4}$ [simplification]

There are no real values for x because $\sqrt{-31}$ is non- real

Worked example:

10. Solve for x by using the quadratic formula

$$2x^2 + 5x - 4 = 0$$

Solution:

$a = 2;\quad b = 5;\quad c = -4$

$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$\boldsymbol{x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-4)}}{2(2)}}$

$x = \frac{-5 \pm \sqrt{57}}{4}$

$x = 0{,}64 \quad or \quad -3{,}14$

Work through all the worked examples until you fully understand each calculation.

Worked example	**CAN YOU?**
12. Solve for x, $(x-3)(x+1) = -2$ **Solution** $(x-3)(x+1) = -2$ [multiply out] $x^2 - 2x - 3 = -2$ [set the equation equal to 0] $x^2 - 2x - 1 = 0$ [this trinomial cannot be factorized] $\boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$ [use the quadratic formula] $\boldsymbol{x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}}$ [substitute a, b and c in the formula] $x = \frac{2 \pm \sqrt{8}}{2}$ [simplify] $\therefore x = 1 + \sqrt{2}$ or $x = 1 - \sqrt{2}$ [solutions in simplest surd form] $\therefore x = 2{,}41$ or $x = -0{,}41$ [solutions correct to two decimal places]	**Solve for x :** 1. $x^2 + 6x - 2 = 0$ 2. $5x^2 - \frac{1}{4}x = 3$ 3. $7(x-3)(x+2) = 6x - 2$ **Answers:** **1.** $x = 0{,}32$ or $x = -6{,}32$ **2.** $x = \frac{3}{4}$ or $x = \frac{4}{5}$ **3.** $x = 3{,}49$ $or - 1.64$

<u>Consolidation</u>

- Equations and inequalities are part of Algebraic expressions which account for $\pm$ 30% of the final Paper 1 examination.
- Solving quadratic equations involving squares and square roots was done in the previous lesson of exponents and Surds
- When a number of decimal places is mentioned, it usually is a clue to use the quadratic formula.
- The roots of an equation are values of the variables that satisfy the equation (make it true).
- Remember to always check each solution to ensure that solutions are valid.

<u>ACTIVITY</u>	**Mind Action Series**	**Via Africa**	**<u>Siyavula</u>**	Classroom mathematics	Platinum
	Pg. 27 – 32 Exercises 4,6 and 7	Mixed Exercise Pg. 23-29 Exercises 1 to 5	End of chapter exercise Pg. 49 – 59 Exercise 7.1, 7.2, 7.3	Pg. 68 Revision exercise	Pg. 51 - 52 Revision exercise