SUBJECT and GRADE	**MATHEMATICS Gr 11**	
TERM 1	*Week 6*	
TOPIC	**EUCLIDEAN GEOMETRY-LESSON 3**	
AIMS OF LESSON	*State and prove the theorems for circle geometry.* • The opposite angles of a cyclic quadrilateral are supplementary. • The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	
RESOURCES	***Paper based resources***	***Digital resources***
	Refer to the chapter on Euclidean Geometry in your textbook.	*Angles in a Cyclic Quadrilateral Proof* *https://www.youtube.com/watch?v=aNLwD4yyL0I* *The exterior angle of a cyclic quadrilateral* *https://www.youtube.com/watch?v=OVmiNWYuejs* *Prove it is a cyclic quadrilateral.* *https://www.youtube.com/watch?v=JkAzebmi_KY* *Circle Theorem Examples* *https://www.youtube.com/watch?v=mT2p4eCfQic*

INTRODUCTION

Circles have different angle properties, described by theorems.

In this lesson we will look at **TWO Theorems** regarding **CYCLIQUADRELATERALS** in a circle:

- The opposite angles of a cyclic quadrilateral are supplementary.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

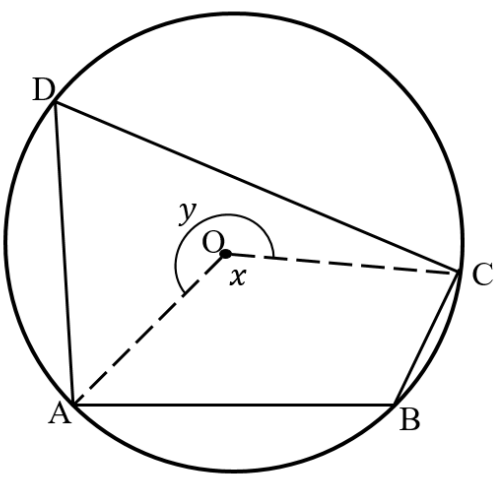
BASIC CIRCLE TERMINOLOGY

- **CYCLIC QUADRILATERALS**

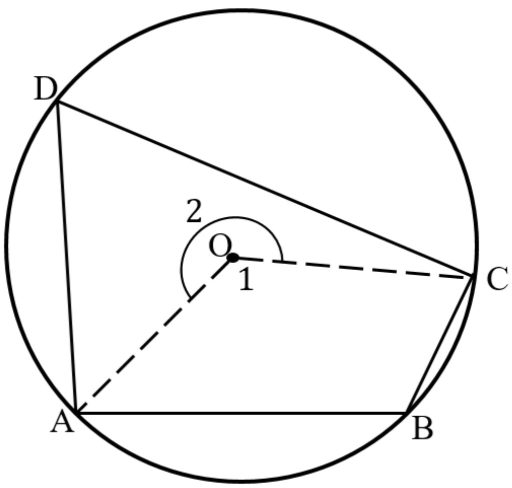
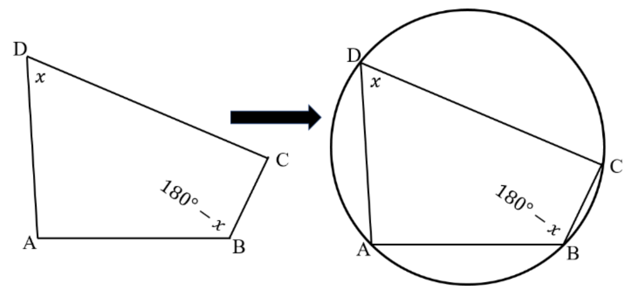
  A quadrilateral whose vertices lie on the **circumference** of a circle is referred to as a **cyclic quadrilateral**.

  ✓ ABCD is a cyclic quadrilateral because A, B, C and D are concyclic.

  × AOCD is NOT a cyclic quadrilateral. *(O is NOT on the circumference of the circle.)*



- **CONCYCLIC**: lie on the circumference. A, B, C and D are concyclic points.

<table>
<tr><td colspan="3">CONCEPTS AND SKILLS</td></tr>
<tr><td colspan="2">THEOREM 6
The opposite angles of a cyclic quadrilateral are supplementary.

$\hat{D} + \hat{B} = 180°$ and $\hat{A} + \hat{C} = 180°$</td><td>CONVERSE THEOREM 6
If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic quadrilateral.
</td></tr>
<tr><td>Acceptable REASON when you use the Theorem in the exam:</td><td>Opp. ∠ s of cyclic quad</td><td>Converse opp. ∠ s of cyclic quad</td></tr>
<tr><td rowspan="2">PROOF OF THEOREM</td><td colspan="2">Given:
A, B, C and D are points that lie on the circumference of the circle.

What to prove: $\hat{D} + \hat{B} = 180°$ and $\hat{A} + \hat{C} = 180°$

Construction: Join AO and CO.</td></tr>
<tr><td colspan="2">Proof:
Let $\hat{D} = x$ and $\hat{B} = y$

$\hat{O}_1 = 2x$ ∠ at centre = 2× ∠ at circumference
$\hat{O}_2 = 2y$ ∠ at centre = 2× ∠ at circumference

$\hat{O}_1 + \hat{O}_2 = 360°$ revolution
$2x + 2y = 360°$
$2(x + y) = 360°$
$x + y = 180°$
$\therefore \hat{D} + \hat{B} = 180°$

Similarly, by joining BO and DO, it can be proven that $\hat{A} + \hat{C} = 180°$</td></tr>
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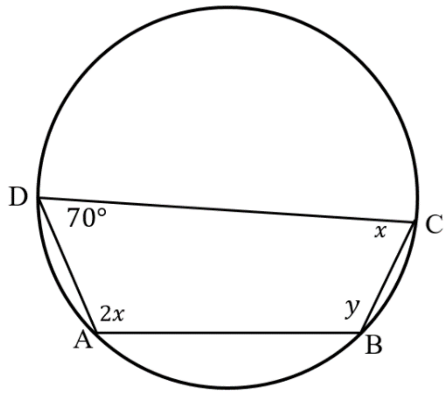
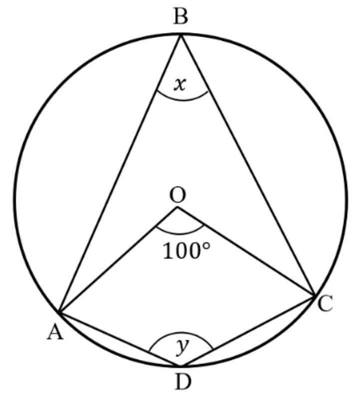
EXAMPLE 1

In the following diagrams, O is the centre of the circle. Determine, with reasons, the value of x and y.

1.1



1.2



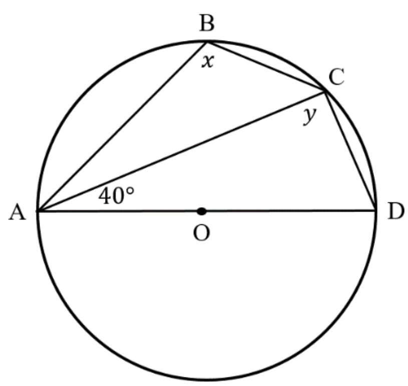
ANSWER:

Statement	***Reason***
1.1 $y = 110°$	Opp. ∠ s of cyclic quad
$2x + x = 180°$ $3x = 180°$ $x = 60°$	Opp. ∠ s of cyclic quad
1.2 $x = 50°$	∠ at centre = 2 × ∠ at circumference
$y = 130°$	Opp. ∠ s of cyclic quad

EXAMPLE 2 - CAN YOU?

In the following diagrams, O is the centre of the circle. Determine, with reasons, the values of x and y.

2.1

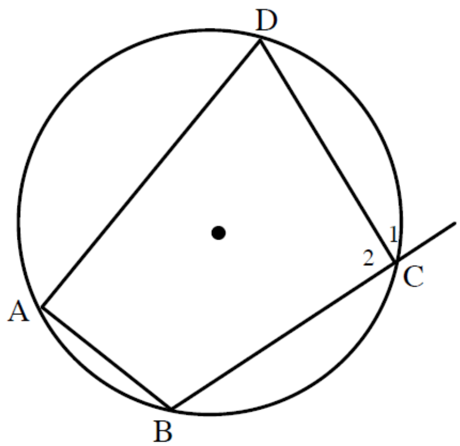


ANSWER:

Statement	***Reason***
2.1 $y = 90°$	∠ in semi−circle
$\widehat{D} = 50°$	Sum ∠s in Δ
$x = 130°$	Opp. ∠ s of cyclic quad

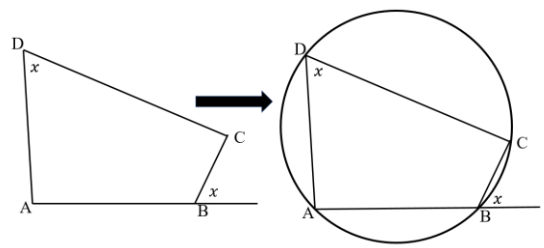
THEOREM 7
An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

You do not have to know the proof for this theorem for exam purposes.



$\widehat{C_1} = \hat{A}$

CONVERSE THEOREM 7
If an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is a cyclic quadrilateral

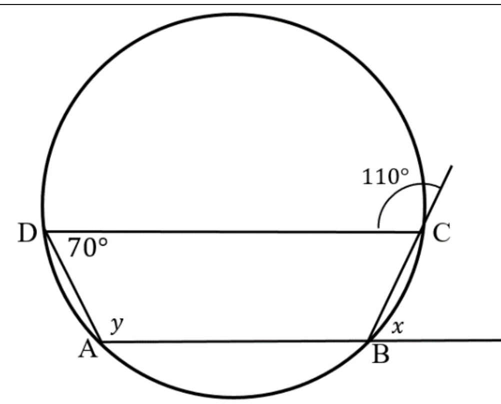


Acceptable REASON when you use the Theorem in the exam:	**Ext. ∠ of cyclic quad**	**Converse ext. ∠ of cyclic quad**

EXAMPLE 3 – CAN YOU?

3.1 Determine, with reasons, the value of x .

3.2 Prove that DC || AB



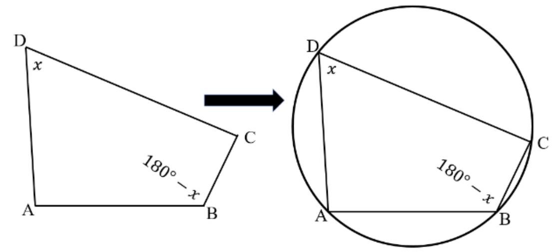
ANSWER:

Statement	***Reason***
3.1 $x = 70°$	Ext.∠ of cyclic quad
3.2 $y = 110°$	Ext.∠ of cyclic quad
$\hat{D} + \hat{A} = 70° + 110°$	
$= 180°$	
∴ AB || CD	Coint ∠s suppl.

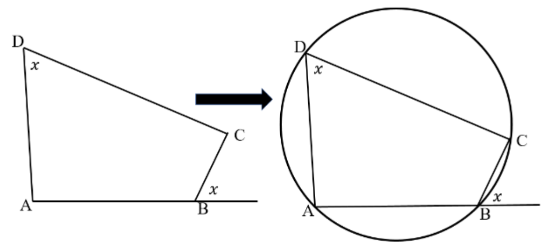
CONCEPTS AND SKILLS

There are **THREE** strategies to proof that a quad is a cyclic quad:

1. *Proof that the opposite angles are supplementary:*

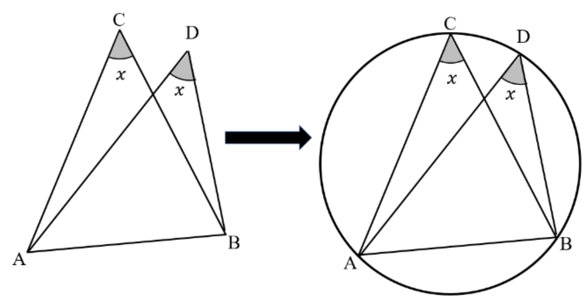


2. *Proof that the exterior angle is equal to the interior opposite angle:*



3. *Proof that two angles subtended by the same line are equal.*

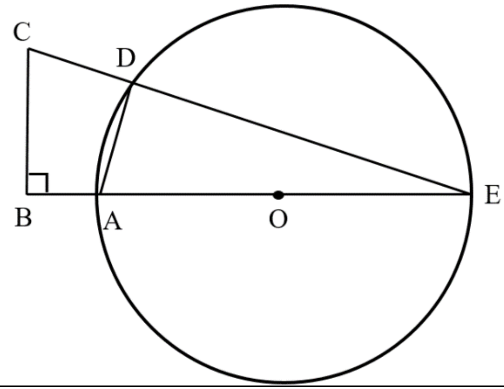
If a line segment joining two points subtends equal angles at two other points on the same side of the line segment, then these four points are concyclic.
(lie on the circumference)



EXAMPLE 4 − (Converse theorems.)

In the diagram is a circle with O the centre of the circle.
CB ⊥ BE

4.1 Prove that ABCD is a cyclic quad.



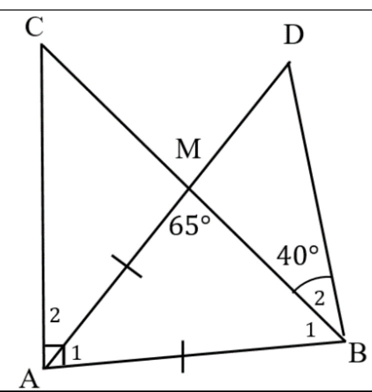
ANSWER:

Statement	*Reason*
4.1 $\widehat{D} = 90°$	∠ in semi − circle
$\therefore \ \widehat{D} = \widehat{B}$	
$\therefore ABCD \text{ is a cyclic quad}$	Converse ext ∠ of cyclic quad

EXAMPLE 5 – CAN YOU?

In the diagram is AM = AB and CA ⊥ AB

5.1 Prove that ABDC is a cyclic quad.



ANSWER:

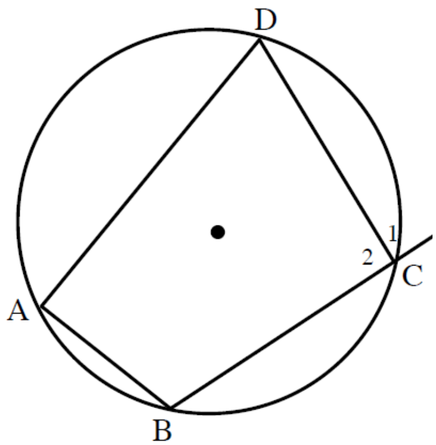
Statement	*Reason*
5.1 $\widehat{B_1} = 65°$	∠ s opp equal sides
$\widehat{A_1} = 50°$	∠ sum in Δ
$\widehat{A_2} = 40°$	Given CA ⊥ AB
$\therefore \ \widehat{A_2} = \widehat{B_2} = 40°$	
$\therefore ABDC \text{ is a cyclic quad}$	*Converse ∠s in same seg.*

ACTIVITIES/ASSESSMENT

MIND ACTION SERIES	CLASSROOM MATHEMATICS	VIA AFRICA
(May 2012 Issue) Chapter 8 • *p* 227 *Exercise* 6 • *p 230 Exercise* 7	• p 266 Exercise 10.3 • p 269 Exercise 10.4	Chapter 8 *p* 219 Exercise 6

CONSOLIDATION

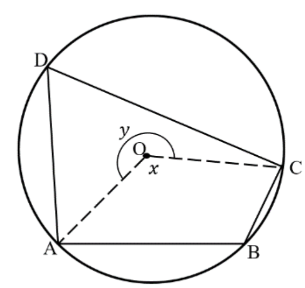
- ***Know and understand the wording of the TWO theorem(s) regarding a cyclic quad.***
- ***Learn the correct way of writing the reason for the Theorem(s)***



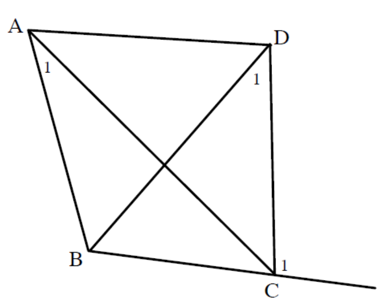
1) $\hat{A} + \widehat{C_2} = 180°$ and $\widehat{D} + \hat{B} = 180°$ (**Opp. ∠s of cyclic quad)**
2) $\hat{A} = \hat{C}1$ (**Ext. ∠ of cyclic quad)**

Recognize when a quad is a cyclic quad and when not:

✓ ABCD is a cyclic quadrilateral because A, B, C and D are concyclic.
× AOCD is NOT a cyclic quadrilateral.
(O is NOT on the circumference of the circle.)



Use these THREE strategies from CONVERSE THEOREMS to proof a quad is a cyclic quad:



- Strategy 1: Proof that B$\hat{A}$D+ B$\hat{C}$D =180°
- Strategy 2: Proof that $\hat{C}_1$= B$\hat{A}$D
- Strategy 3: Proof that $\hat{A}_1 = \widehat{D}_1$