SUBJECT and GRADE	**MATHEMATICS Gr 11**	
TERM 1	*Week 7*	
TOPIC	**Tangent Theorems**	
AIMS OF LESSON	*State and prove the theorems for circle geometry.* • A tangent to a circle is perpendicular to the radius at the point of contact. • The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. • Two tangents drawn to a circle from the same point outside the circle are equal in length	
RESOURCES	***Paper based resources***	***Digital resources***
	Refer to the chapter in your textbooks on Circle Geometry.	*Proof of the tan-chord theorem:* *https://www.youtube.com/watch?v=mjRqu3oJtfA* *Understand Alternate Segment Theorem to find equal angles in Circle* *https://www.youtube.com/watch?v=IvfIxezwb5A* *https://www.youtube.com/watch?v=QmPUlc5BDmk* *Tan; radius theorem:* *https://www.youtube.com/watch?v=IcgycGSq9Us* *Tan; radius theorem and tans from the same point theorem:* *https://www.youtube.com/watch?v=nQntUl7Wbe0* *All three tangent theorems in one:* *https://www.youtube.com/watch?v=DroUzFiqRsc*

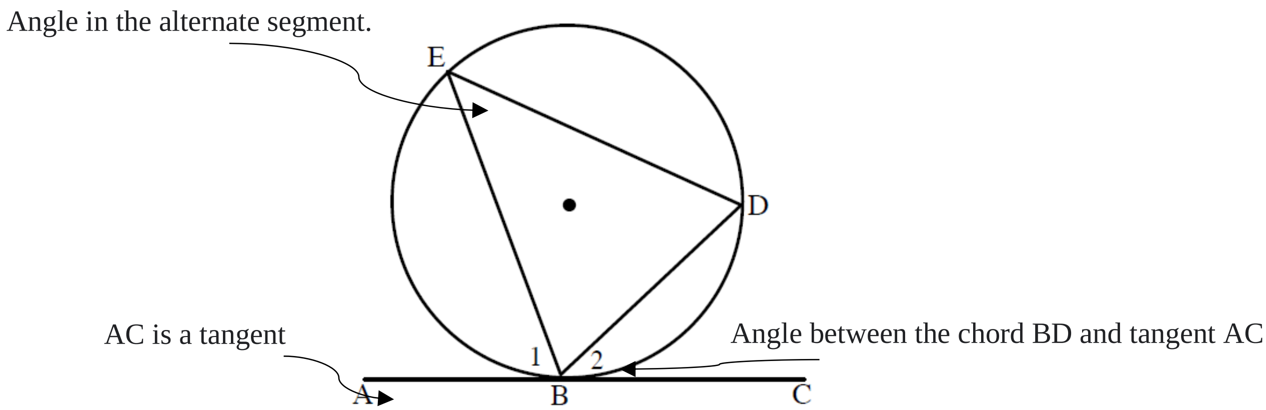
INTRODUCTION

Circles have different angle properties, described by theorems.
In this lesson we will look at **THREE Theorems** regarding **TANGENTS** to a circle:

- A tangent to a circle is perpendicular to the radius at the point of contact.
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.
- Two tangents drawn to a circle from the same point outside the circle are equal in length.

BASIC CIRCLE TERMINOLOGY

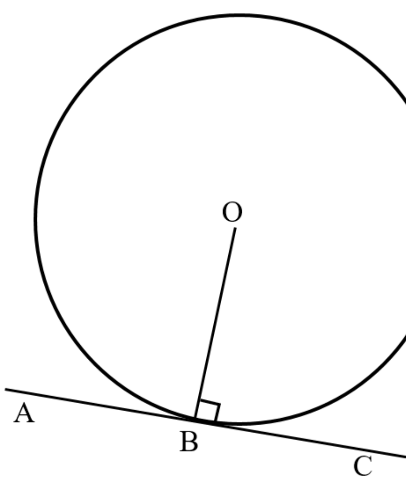
- **TANGENT:** The **tangent to a circle** is defined as a straight line which touches the **circle** at a single point. The point where the **tangent** touches a **circle** is known as the point of **tangency** or the point of contact.



CONCEPTS AND SKILLS

THEOREM 8

A tangent to a circle is perpendicular to the radius at the point of contact.



If AC is a tangent and OB is a radius then OB⊥AC

CONVERSE THEOREM 8

If a line is drawn perpendicular to a radius at the point where the radius meets the circle,
then the line is a tangent to the circle.

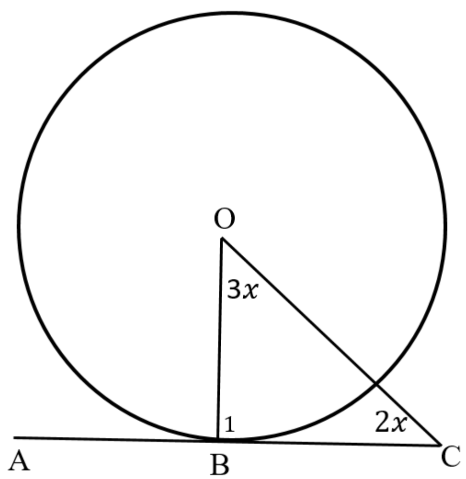
You do not have to know the proof for these theorems for exam purposes.

Acceptable REASON when you use the Theorem in the exam:	**tan ⊥ radius**	**Converse tan ⊥ radius**

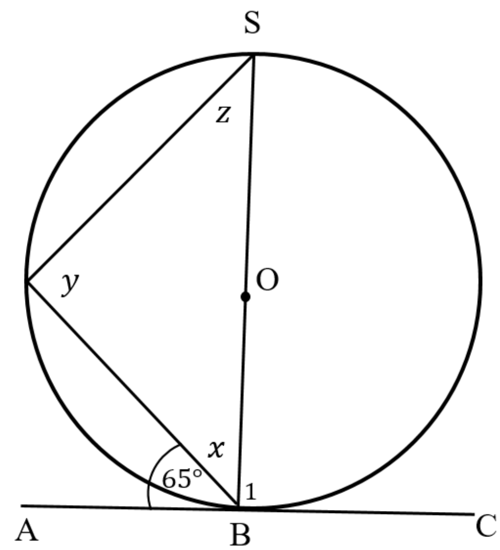
EXAMPLE 1

In the following diagrams, O is the centre of the circle. Determine, with reasons, the value of x ; y *and* z.

1.1



1.2 − CAN YOU?



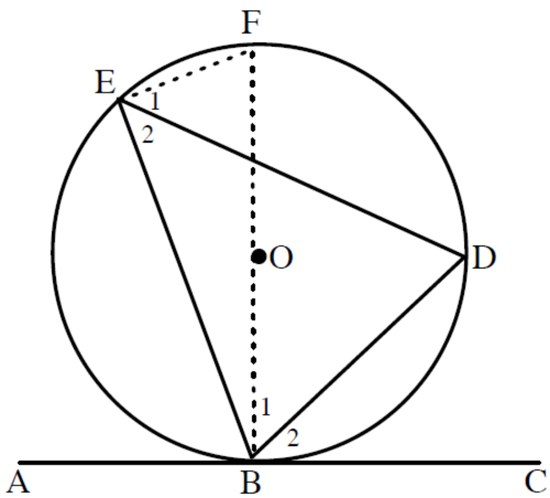
ANSWER:

Statement	***Reason***
1.1 $\widehat{B_1} = 90°$	tan ⊥ radius
$2x + 3x + 90° = 180°$	sum ∠ s in Δ
$5x = 90°$	
$x = 18°$	
1.2 $x = 25°$	tan ⊥ radius
$y = 90°$	∠ in semi−circle
$z = 65°$	sum ∠ s in Δ

CONCEPTS AND SKILLS

THEOREM 9

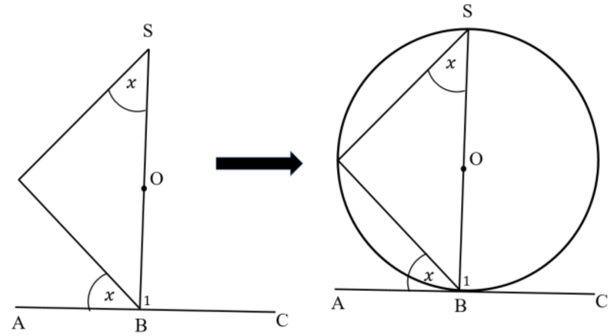
The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.



$$\widehat{B_2} = \widehat{E_2} \quad and \ A\hat{B}E = \hat{D}$$

CONVERSE THEOREM 9

If a line is drawn through the endpoint of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.



Acceptable REASON when you use the Theorem in the exam:	Tan-chord theorem	Converse tan-chord theorem

PROOF OF THEOREM

Given:
Tangent ABC

What to prove: $\widehat{B_2} = \widehat{E_2}$

Construction: Draw diameter BOF and join EF

Proof:

$\hat{B}_1 + \hat{B}_2 = 90°$ tan ⊥ radius
$\hat{E}_1 + \hat{E}_2 = 90°$ ∠ in semi-circle

Let $\widehat{\mathrm{B}_1} = x$
$\therefore \hat{B}_2 = 90° - x$

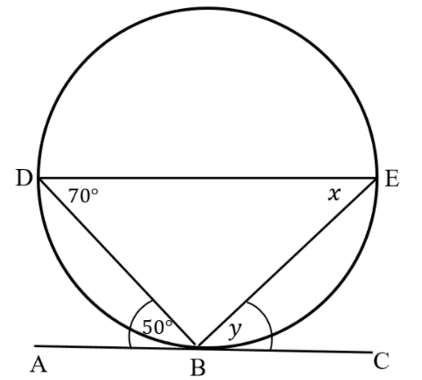
$\hat{B}_1 = \hat{E}_1 = x$ ∠ s in the same segment
$\therefore \hat{E}_2 = 90° - x$

$\therefore \widehat{B_2} = \widehat{E_2}$

EXAMPLE 2

In the diagram is AC a tangent to the circle at point B.

2.1 Determine, with reasons, the value of $x\ and\ y$.



ANSWER:

Statement	***Reason***
2.1 $x = 50°$	Tan-chord theorem
$y = 70°$	Tan-chord theorem

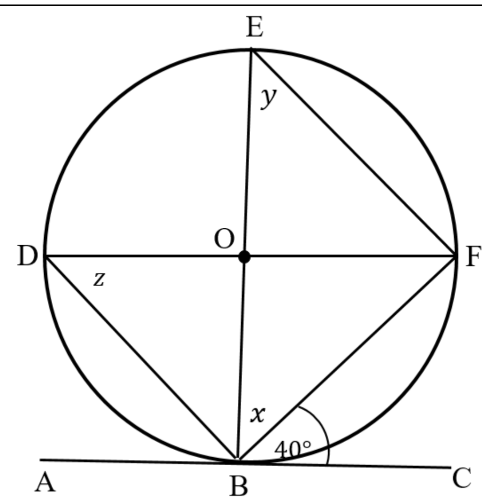
EXAMPLE 3– CAN YOU?

In the diagram AC is a tangent to the circle at point B. O is the centre of the circle.

3.1 Determine, with reasons, the value of z.

3.2 Determine, with reasons, the value of x .

3.3 Determine, with reasons, the value of y .



ANSWER:

Statement	***Reason***
3.1 $z = 40°$	Tan-chord theorem
3.2 $x = 50°$	tan ⊥ radius
3.3 $y = 40°$	Tan-chord theorem

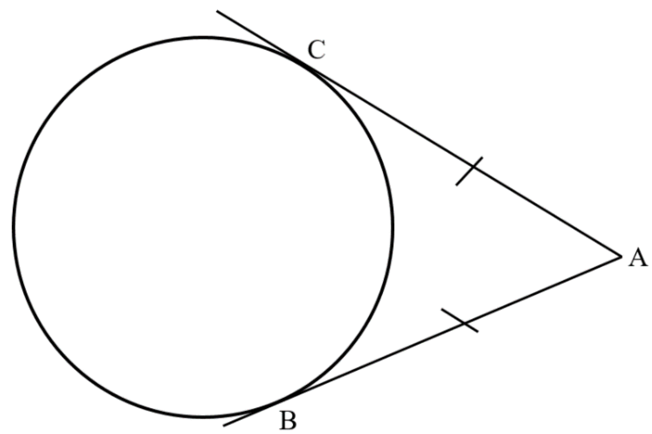
CONCEPTS AND SKILLS

THEOREM 10

If two tangents are drawn from the same point outside a circle, then they are equal in length.

You do not have to know the proof for these theorems for exam purposes.

CAN YOU THINK OF ONE?



If AC is a tangent and AB is a tangent then AC = AB

Acceptable REASON when you use the Theorem in the exam:	**Tans from the same point A**

EXAMPLE 4

In the diagram AC and AB are tangents to the circle at point C and B. O is the centre of the circle.

4.1 Determine, with reasons, the value of x and y

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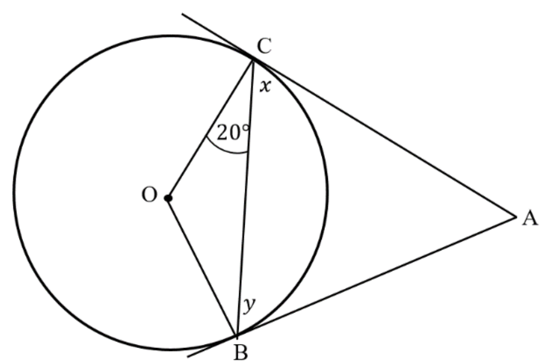


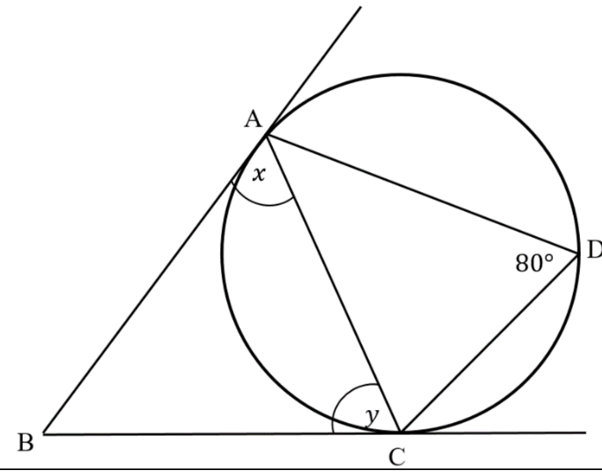
ANSWER:

Statement	***Reason***
4.1 AC = AB	tans from the same point A
$x = 70°$	tan ⊥ radius
$y = 70°$	∠ s opp equal tans.

EXAMPLE 5– CAN YOU?

In the diagram AB and BC are tangents to the circle at point A and C.

5.1 Determine, with reasons, the value of x and y.



ANSWER:

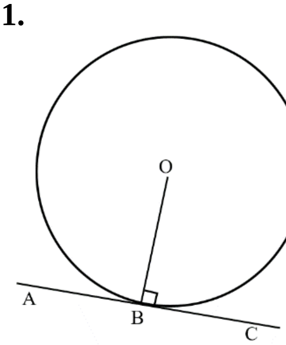
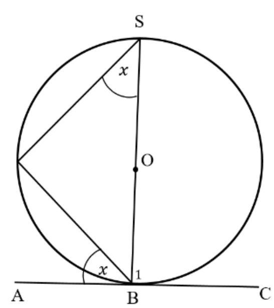
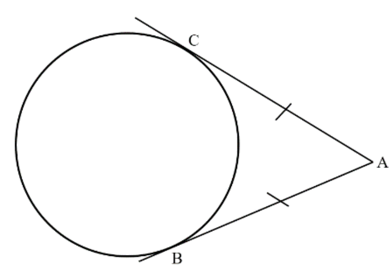
Statement	***Reason***
5.1 AB = BC	tans from the same point B.
$x = 80°$	tan-chord theorem
$y = 80°$	$\angle$ s opp equal tans.

ACTIVITIES/ASSESSMENT

MIND ACTION SERIES (May 2012 Issue)	CLASSROOM MATHEMATICS	VIA AFRICA
Chapter 8 • *p* 234 *Exercise* 8 • *p 236 Exercise* 9	• p 275 Exercise 10.6 • p 277 Exercise 10.7	Chapter 8 • *p* 222 Exercise 8

CONSOLIDATION

- ***Know and understand the wording of the TWO theorem(s) regarding a cyclic quad.***
- ***Learn the correct way of writing the reason for the Theorem(s)***

1.	**2.**	**3.**
		
tan ⊥ radius	**Tan-chord theorem**	**tans from the same point**

<table>
<tr><td colspan="2">There are TWO strategies to proof that a line is a tangent:</td></tr>
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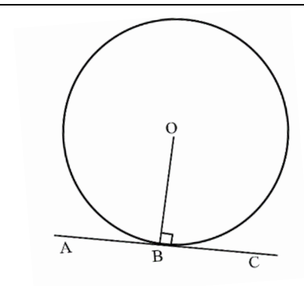
1. *Proof that the line is drawn perpendicular to a radius at the point where the radius meets the circle, then the line is a tangent to the circle.*



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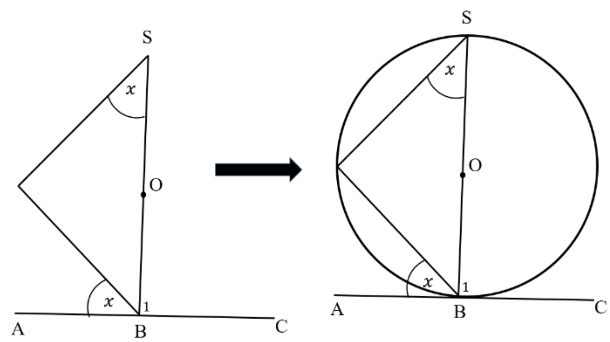
2. *Proof that the line drawn through the endpoint of a chord, makes with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.:*



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<tr><td>VALUES</td><td>

A line is *tangent* to a circle if it touches it at one and only one point. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. Check out the bicycle wheels in the below figure.

In this figure, the wheels are, of course, circles, the spokes are radii, and the ground is a *tangent line*. The point where each wheel touches the ground is a *point of tangency*. And the most important thing — what the theorem tells you — is that the radius that goes to the point of tangency is *perpendicular* to the tangent line.

https://www.dummies.com/education/math/geometry/how-a-tangent-relates-to-a-circle/

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