



SUBJECT and GRADE	MATHEMATICS GR 11		
TERM 2	Week 4		
TOPIC	FUNCTIONS		
AIMS OF LESSON	<ul style="list-style-type: none"> <li>• Sketching the graphs of a parabolic-, hyperbolic- and exponential function.</li> <li>• Introducing the Parabola equations in turning point form and standard form.</li> <li>• Use concepts of shape, turning point, axes of symmetry, intercepts and asymptotes as they relate to the graphs.</li> </ul>		
RESOURCES	<i>Paper based resources</i>	<i>Digital resources</i>	
	Please go to the Functions section in your Mathematics Textbook.	Parabola: <a href="https://bit.ly/2KMIuwW">https://bit.ly/2KMIuwW</a> ; <a href="https://bit.ly/2ycgLTF">https://bit.ly/2ycgLTF</a> ; <a href="https://bit.ly/3eWbfFk">https://bit.ly/3eWbfFk</a> Hyperbola: <a href="https://bit.ly/3aQokg4">https://bit.ly/3aQokg4</a> ; <a href="https://bit.ly/3aOhEiz">https://bit.ly/3aOhEiz</a> ; <a href="https://bit.ly/3bSa8Va">https://bit.ly/3bSa8Va</a> Exponential: <a href="https://bit.ly/2KLNbxq">https://bit.ly/2KLNbxq</a>	
INTRODUCTION	<p>In Grade 10 you have learnt to sketch graphs of the form:  <math>y = ax^2 + q</math>, <math>y = \frac{a}{x} + q</math> and <math>y = a \cdot b^x + q</math></p> <p>Here are some examples of graphs sketched in GR10.</p>		
• <b>Standard form</b>	$y = ax^2 + q$	$y = \frac{a}{x} + q$	$y = a \cdot b^x + q$
• Equation for this example	$f(x) = x^2 - 9$	$g(x) = \frac{9}{x} + 4$	$h(x) = 2^x - 2$
• <b>x –intercepts (y = 0)</b>	$(-3; 0)$ and $(3; 0)$	$(-\frac{9}{4}; 0)$	$(1; 0)$
• <b>y –intercept (x = 0)</b>	$(0; 9)$	none	$(0; -1)$
• <b>Asymptotes</b>		horizontal asymptote $y = 4$ vertical asymptote $x = 0$	horizontal asymptote $y = -2$
• <b>Other</b>	$a > 0 \rightarrow$ arms up	$k > 0 \rightarrow$ graph in 1st and 3rd quadrant	$b > 1 \rightarrow$ graph is increasing



**CONCEPTS AND SKILLS**

**(1) PARABOLA**

In this lesson we are going onto sketching the same graphs, but we will include an extra parameter  $p$ .

$$y = a(x + p)^2 + q, y = \frac{k}{x+p} + q \text{ and } y = a \cdot b^{x+p} + q$$

We will discover that this variable will translate (move) the graphs left or right.

**HOW TO SKETCH THE GRAPH:**  $y = a(x + p)^2 + q$

There are **THREE** concepts that the examiner will award marks for:

- Turning point
- $x$  –intercepts
- $y$  – intercept

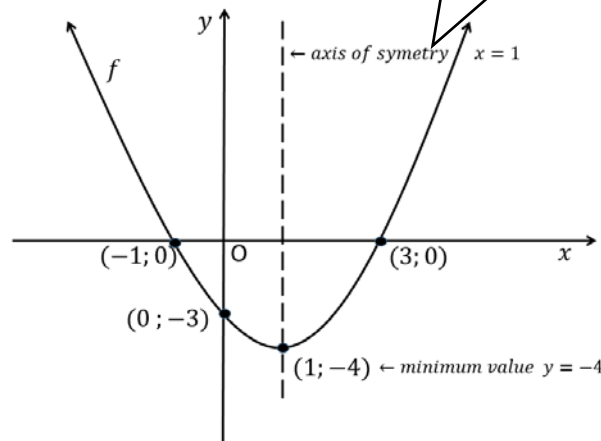
**EXAMPLE 1**

**1.1 Draw the sketch**  $f(x) = (x - 1)^2 - 4$

Clearly show all intercepts with the axes and the coordinates of the turning point.

**ANSWER:**

- $a > 0 \rightarrow$  arms up
- **TP**  $(1; -4)$
- **$x$  –intercepts ( $y = 0$ )**  
 $0 = (x - 1)^2 - 4$   
 $0 = x^2 - 2x + 1 - 4$   
 $0 = x^2 - 2x - 3$   
 $0 = (x - 3)(x + 1)$   
 $\therefore x = 3 ; x = -1$
- **$y$  –intercept ( $x = 0$ )**  
 $y = (0 - 1)^2 - 4$   
 $y = -3$



It is always a good idea to indicate the axis of symmetry with a broken line

**1.2 Does this graph have a minimum or a maximum value? Give the value.**

**ANSWER:**

Minimum value of  $y$  is  $-4$

**NOTE:**

**$f(x)$  –notation:**

To distinguish between different graphs, we can name them with letters of the alphabet. For this reason, we use  $f(x), g(x), h(x)$  etc. in the place of the  $y$  –value.

**CAN YOU?**

- 1) Draw the sketch  $g(x) = (x + 2)^2 - 9$   
Clearly show all intercepts with the axes and the coordinates of the turning point.
- 2) Does this graph have a minimum or a maximum value? Give the value.
- 3) Give the equation for the axis of symmetry.

**ANSWER:**

1)

**$x$  –intercepts ( $y = 0$ )**

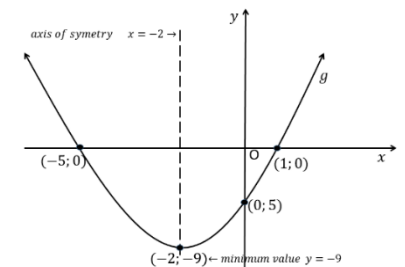
$$0 = (x + 2)^2 - 9$$

$$0 = x^2 + 4x + 4 - 9$$

$$0 = x^2 + 4x - 5$$

$$0 = (x + 5)(x - 1)$$

$$x = -5 ; x = 1$$



- 2) minimum value of  $y$  is  $-9$
- 3)  $x = -2$



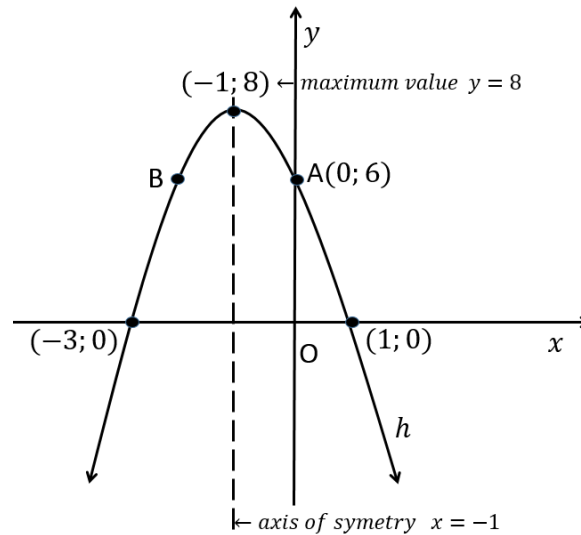
**EXAMPLE 2**

**2.1 Draw the sketch  $h(x) = -2(x + 1)^2 + 8$**

**Clearly show all intercepts with the axes and the turning point.**

**ANSWER:**

- $a < 0 \rightarrow$  arms down
- **TP  $(-1; 8)$**
- **$x$ -intercept ( $y = 0$ )**  
 $0 = -2(x + 1)^2 + 8$   
 $0 = -2(x^2 + 2x + 1) + 8$   
 $0 = -2x^2 - 4x - 2 + 8$   
 $0 = -2x^2 - 4x + 6$   
 $0 = -2(x^2 + 2x - 3)$   
 $0 = -2(x + 3)(x - 1)$   
 $\therefore x = -3 ; x = 1$
- **$y$ -intercept ( $x = 0$ )**  
 $y = -2(0 + 1)^2 + 8$   
 $y = 6$



**Symmetry in sketch:**

**2.2 Give the coordinates of B, if B is the reflections of A(0; 6) around the axes of symmetry.**

**ANSWER:**

**B(-2; 6)**

**NOTE:**

The  $-2$  will NOT influence the coordinates of the turning point

**CAN YOU?**

4) Draw the sketch  $k(x) = 2(x - 2)^2 - 8$

Clearly show all intercepts with the axes and the coordinates of the turning point.

5) Give the coordinates of C, if C is the reflections of D(1; -6) around the axes of symmetry.

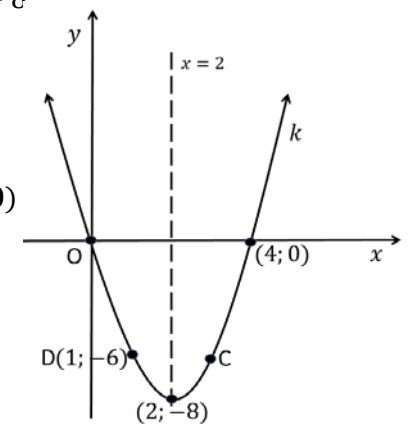
**ANSWER:**

4)

- $a > 0 \rightarrow$  arms up
- **TP  $(2; -8)$**
- **$x$ -intercept ( $y = 0$ )**  
 $0 = 2(x - 2)^2 - 8$   
 $0 = 2(x^2 - 4x + 4) - 8$   
 $0 = 2x^2 - 8x + 8 - 8$   
 $0 = 2x^2 - 8x$   
 $0 = 2x(x - 4)$   
 $\therefore x = 0 ; x = 4$

- **$y$ -intercept ( $x = 0$ )**  
 $y = 2(0 - 2)^2 - 8$   
 $y = 0$

5) C(3; -6)





NOTE:  
This equation is in standard form.

We cannot “see” the turning point in this form.

We must use the following formula to get the symmetry axis:

$$x = \frac{-b}{2a}$$

**EXAMPLE 3**

**3.1 Draw the sketch  $f(x) = -2x^2 - 4x + 6$**

Clearly show all intercepts with the axis and the turning point.

**ANSWER:**

- $a < 0 \rightarrow$  arms down
- **TP**

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-4)}{2(-2)}$$

$$x = -1$$

Substitute this value into the ORIGINAL EQUATION to calculate the y –coordinate.

$$f(-1) = -2(-1)^2 - 4(-1) + 6$$

$$f(-1) = 8$$

**Turning Point  $(-1; 8)$**

- **x –intercepts  $(y = 0)$**

$$0 = -2x^2 - 4x + 6$$

$$0 = -2(x^2 + 2x - 3)$$

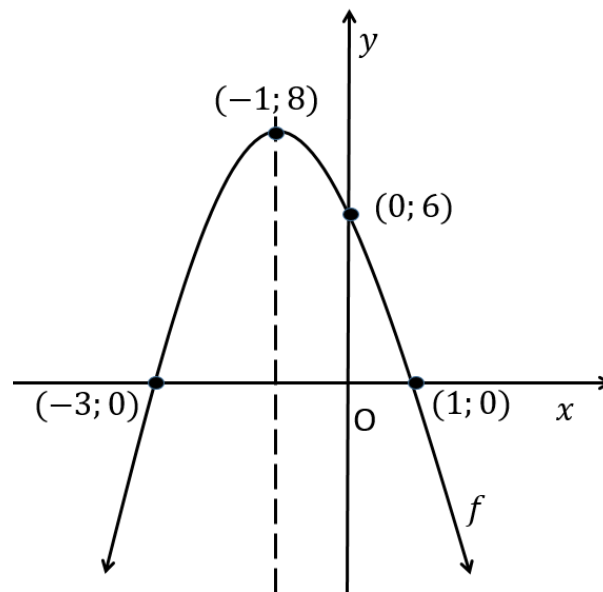
$$0 = -2(x + 3)(x - 1)$$

$$\therefore x = -3 ; x = 1$$

- **y –intercept  $(x = 0)$**

$$y = -2(0)^2 - 4(0) + 6$$

$$y = 6$$



**CAN YOU?**

**6) Draw the sketch  $g(x) = x^2 - 8x + 16$**

Clearly show all intercepts with the axis and the coordinates of the turning point

**ANSWER:**

- $a > 0 \rightarrow$  arms up

- **TP**

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)}$$

$$x = 4$$

$$f(4) = (4)^2 - 8(4) + 16$$

$$f(4) = 0$$

**TP  $(4; 0)$**

- **x –intercepts  $(y = 0)$**

$$0 = x^2 - 8x + 16$$

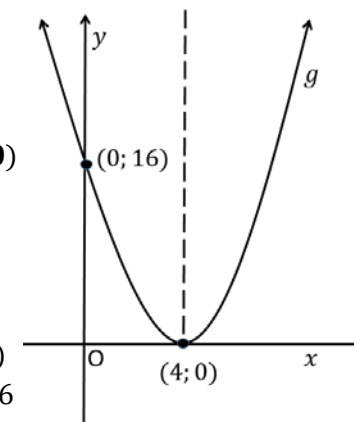
$$0 = (x - 4)(x - 4)$$

$$\therefore x = 4 ; x = 4$$

- **y –intercept  $(x = 0)$**

$$y = (0)^2 - 8(0) + 16$$

$$y = 16$$





**CONCEPTS AND SKILLS**

**(2) HYPERBOLA**

**HOW TO SKETCH THE GRAPH:**  $y = \frac{a}{x+p} + q$

There are **THREE** concepts that the examiner will award marks for:

- Horizontal and vertical asymptotes.
- $x$  –intercept
- $y$  – intercept

**EXAMPLE 4**

4.1) Sketch the graph of  $h(x) = \frac{8}{x+1} - 4$  clearly show all asymptotes and the intercepts with the axes.

**ANSWER:**

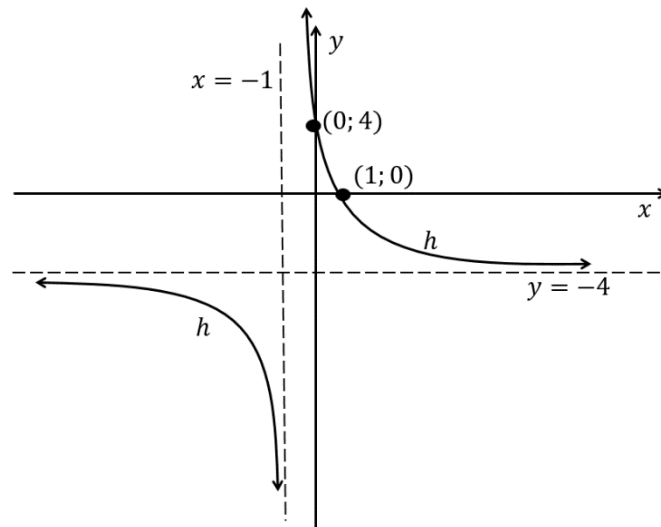
- Horizontal asymptote:  $y = -4$  and vertical asymptotes:  $x = -1$

- $x$  –intercept ( $y = 0$ )

$$\begin{aligned} 0 &= \frac{8}{x+1} - 4 \\ 4 &= \frac{8}{x+1} \\ 4(x+1) &= 8 \\ 4x+4 &= 8 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

- $y$  – intercept

$$\begin{aligned} y &= \frac{8}{0+1} - 4 \\ y &= 4 \end{aligned}$$



**CAN YOU?**

7) Sketch the graph of  $p(x) = \frac{-4}{x-1} - 2$

clearly show all asymptotes and the intercepts with the axes.

**ANSWER:**

- Horizontal asymptote:  $y = -2$

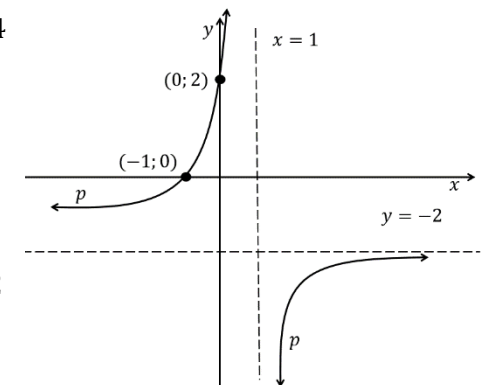
- vertical asymptote:  $x = 1$

- $x$  –intercept ( $y = 0$ )

$$\begin{aligned} 0 &= \frac{-4}{x-1} - 2 \\ 2 &= \frac{-4}{x-1} \\ 2(x-1) &= -4 \\ 2x-2 &= -4 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

- $y$  – intercept

$$\begin{aligned} y &= \frac{-4}{0-1} - 2 \\ y &= 2 \end{aligned}$$





**CONCEPTS AND SKILLS**

**(2) EXPONENTIAL GRAPH**

**HOW TO SKETCH THE GRAPH:**  $y = a \cdot b^{x+p} + q$

There are **THREE** concepts that the examiner will award marks for:

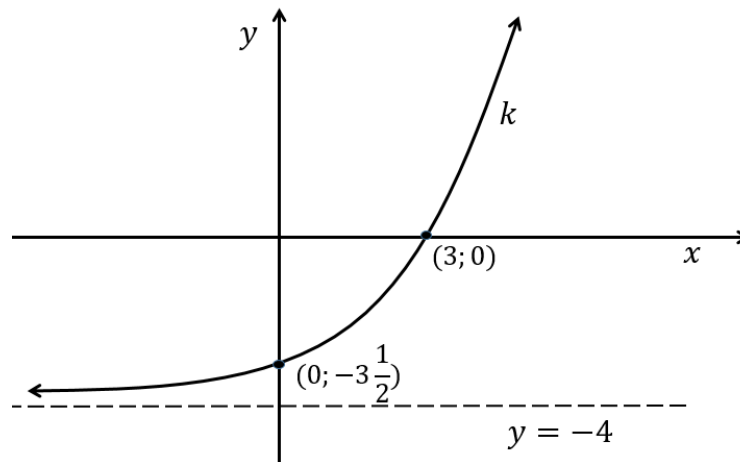
- Horizontal asymptote.
- $x$  –intercept
- $y$  – intercept

**EXAMPLE 5**

5.1 Sketch the graph of  $k(x) = 2^{x-1} - 4$  clearly show all asymptote(s) and the intercepts with the axes.

**ANSWER:**

- Horizontal asymptote:  $y = -4$
- $x$  –intercept ( $y = 0$ )  
 $0 = 2^{x-1} - 4$   
 $4 = 2^{x-1}$   
 $2^2 = 2^{x-1}$   
 $2 = x - 1$   
 $3 = x$
- $y$  – intercept  
 $y = 2^{0-1} - 4$   
 $y = \frac{1}{2} - 4$   
 $y = -3\frac{1}{2}$

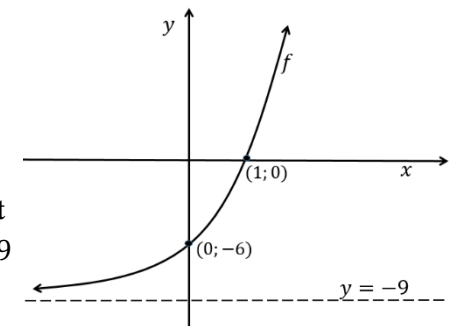


**CAN YOU?**

8) Sketch the graph of  $f(x) = 3^{x+1} - 9$  clearly show all asymptotes and the intercepts with the axes.

**ANSWER:**

- Horizontal asymptote:  $y = -9$
- $x$  –intercept ( $y = 0$ )  
 $0 = 3^{x+1} - 9$   
 $9 = 3^{x+1}$   
 $3^2 = 3^{x+1}$   
 $2 = x + 1$   
 $1 = x$
- $y$  – intercept  
 $y = 3^{0+1} - 9$   
 $y = 3 - 9$   
 $y = -6$



9) Give the equation for  $g$  if  $g$  is the graph after  $f$  has moved 6 units up and 1 unit to the left.

**ANSWER:**

$g(x) = 3^{x+2} - 3$



**HOW TO SKETCH THE GRAPH:**  $y = a \cdot b^{x+p} + q$

**EXAMPLE 6**

6.1) Sketch the graph of  $k(x) = \frac{1}{2}^{x-2} - 8$  clearly show all asymptote(s) and the intercepts with the axes.

**ANSWER:**

- **Horizontal asymptote:**  $y = -8$

- **x-intercept ( $y = 0$ )**

$$0 = \frac{1}{2}^{x-2} - 8$$

$$8 = (2^{-1})^{x-2}$$

$$2^3 = 2^{-x+2}$$

$$3 = -x + 2$$

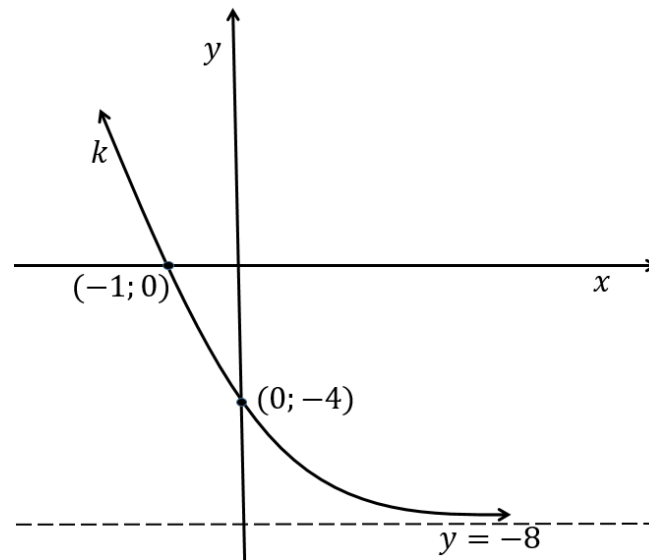
$$x = -1$$

- **y-intercept**

$$y = \frac{1}{2}^{0-2} - 8$$

$$y = 4 - 8$$

$$y = -4$$



**CAN YOU?**

10) Sketch the graph of  $g(x) = \frac{1}{5}^{x+1} - 1$  clearly show all asymptotes and the intercepts with the axes.

**ANSWER:**

- **Horizontal asymptote:**  $y = -1$

- **x-intercept ( $y = 0$ )**

$$0 = \frac{1}{5}^{x+1} - 1$$

$$1 = \frac{1}{5}^{x+1}$$

$$5^0 = (5^{-1})^{x+1}$$

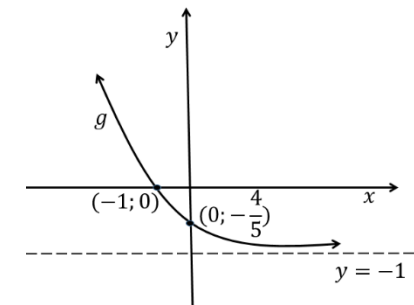
$$0 = -x - 1$$

$$x = -1$$

- **y-intercept**

$$y = \frac{1}{5}^{0+1} - 1$$

$$y = -\frac{4}{5}$$



11) Give the equation for  $h$  if  $h$  is the graph after  $f$  has moved 1 units up and 1 unit to the right.

**ANSWER:**

$$h(x) = \frac{1}{5}^x$$

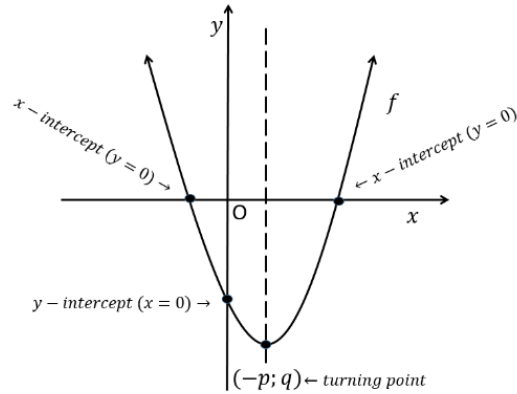
**ACTIVITIES/ASSESSMENT**

- VIA AFRICA p 52 – 56 Question 1 – 9
- MIND ACTION SERIES Exercise 1 – 6 p 87 – 114
- SIYAVULA Exercise 5 – 4 p154 ; Exercise 5 – 5 p155 and Exercise 5 – 6 p161
- PLATINUM Exercise 3 p 86 ; Exercise 4 p 89 en Exercise 7 p 97



**CONSOLIDATION**

**PARABOLA**  $y = a(x + p)^2 + q$

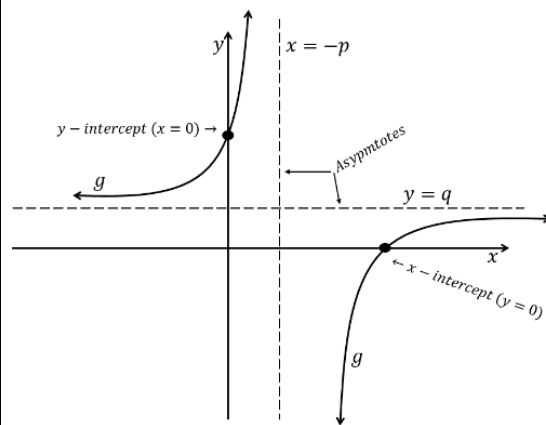


- $x$  - intercept ( $y = 0$ ) &  $y$  - intercept ( $x = 0$ )
- Parabola: Turning point  $(-p; q)$

	$p > 0$		$p < 0$	
	$a < 0$	$a > 0$	$a < 0$	$a > 0$
$q > 0$				
$q < 0$				

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**HYPERBOLA**  $y = \frac{a}{x+p} + q$



- $x$  - intercept ( $y = 0$ ) &  $y$  - intercept ( $x = 0$ )
- Asymptotes  $x = -p$  and  $y = q$

	$p > 0$		$p < 0$	
	$a < 0$	$a > 0$	$a < 0$	$a > 0$
$q > 0$				
$q < 0$				

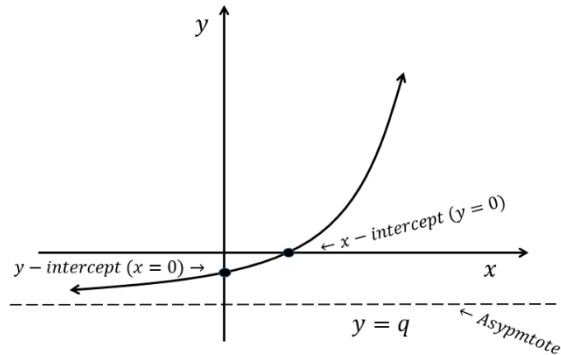
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**ESPONENTIAL GRAPH:**

$y = a \cdot b^{x+p} + q$



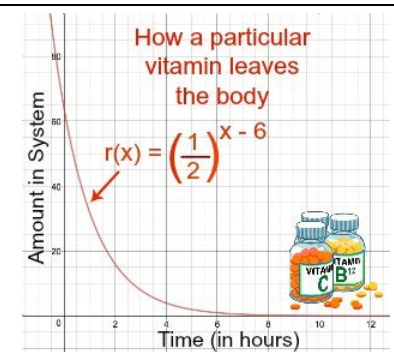
- $x - intercept (y = 0)$  &  $y - intercept (x = 0)$
- Asymptote  $y = q$

$b > 1$	$a < 0$	$a > 0$	$0 < b < 1$	$a < 0$	$a > 0$
$q > 0$			$q > 0$		
$q < 0$			$q < 0$		

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**VALUES:**

*Concepts/skills in real life scenarios*



<http://passyworldofmathematics.com/sydney-harbour-bridge-mathematics/>

<https://study.com/academy/lesson/modeling-the-real-world-with-families-of-functions.html>