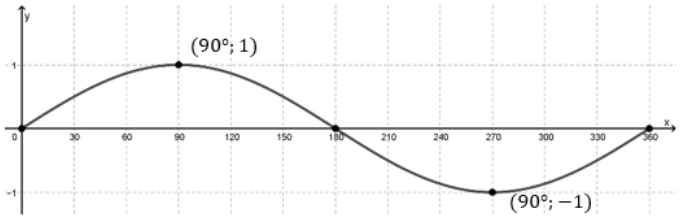
SUBJECT and GRADE	Mathematics Grade 11	
TERM 3	*Week 4*	
TOPIC	Trigonometrical Functions	
AIMS OF LESSON	• To determine the effect of k and p on the trigonometrical functions: $y = \sin kx$ and $y = \sin(x + p)$ • Draw graphs with different periods and horizontal shifts • Interpret trigonometrical functions	
RESOURCES	***Paper based resources***	***Digital resources***
	Refer to Trigonometric Graphs in your textbook.	https://www.youtube.com/watch?v=bRSrA3Wf8FI https://www.youtube.com/watch?v=qFhcKqOXyQI

INTRODUCTION: $y = a\sin x + q$; $y = a\cos x + q$ and $y = a\tan x + q$

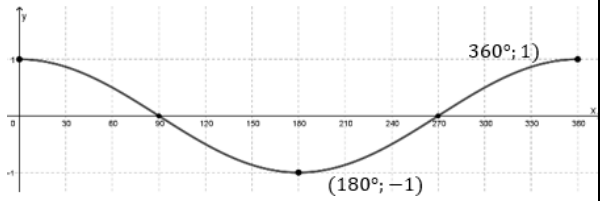
The focus in Grade 10 was the impact of a and q on the trigonometric graph. We discovered that, a influences the amplitude while $+ q$ moves the graph q units up and $-q$ moves the graph q units down.

The three basic graphs:

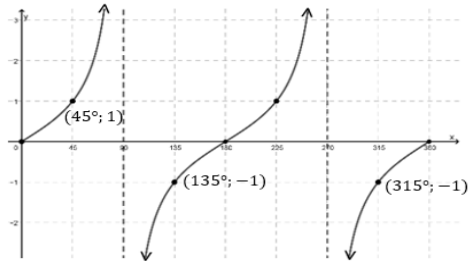
$y = \sin x$



$y = \cos x$



$y = \tan x$

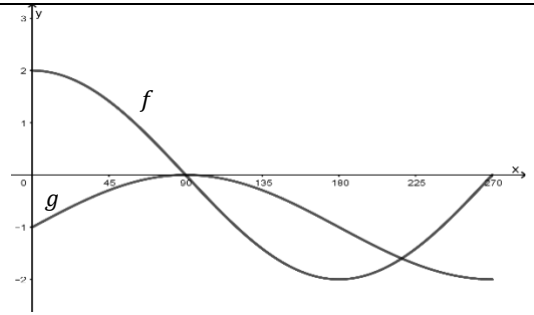


How much can you remember?

In the adjacent diagram are the graphs of:

$f(x) = a\cos x$ and $g(x) = b\sin x - q$

1. Write down the amplitude g.
2. Write down the values a, b and q.
3. The range of g.



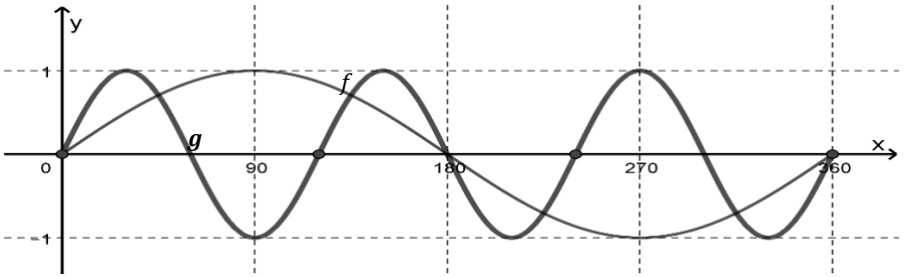
Solutions:

1. amplitude: 1
2. $a = 2$, $b = 1$ and $q = 1$.
3. The range of g: $y \in [-2; 0]$ for $y \in \mathbb{R}$

CONCEPTS AND SKILLS

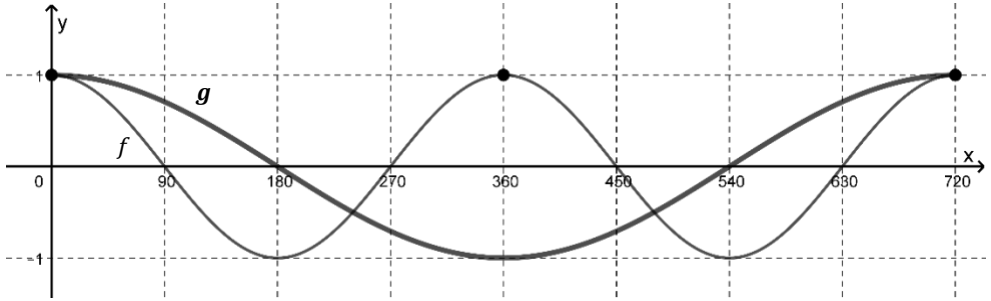
We will be exploring the effect of k in $y = \sin kx$ and $y = \cos kx$. Study the graphs below. What differences do you note?

$f(x) = \sin x$ and $g(x) = \sin 3x$
for $x \in [0°; 360°]$.



There are 3 cycles within the normal (360°) period.
The period of $g(x)$ is 120°

$f(x) = \cos x$ and $g(x) = \cos\frac{1}{2}x$
for $x \in [0°; 720°]$



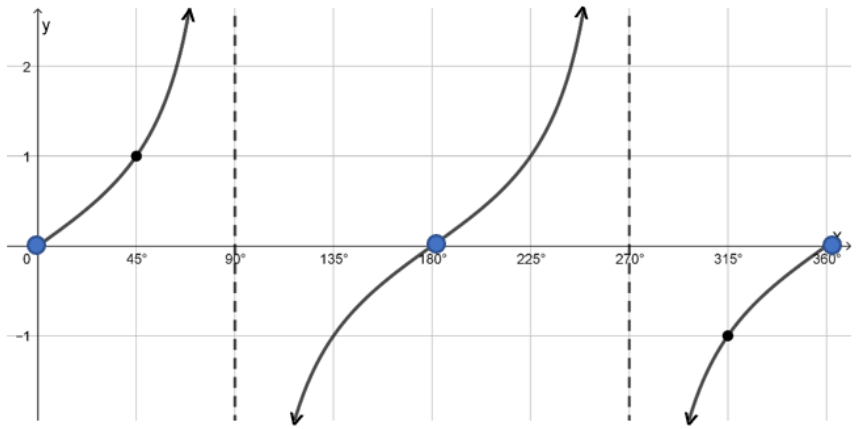
$g(x)$ is **half** of the normal period.
The period of $g(x)$ will end at 720°

Complete the table below for the above graphs:

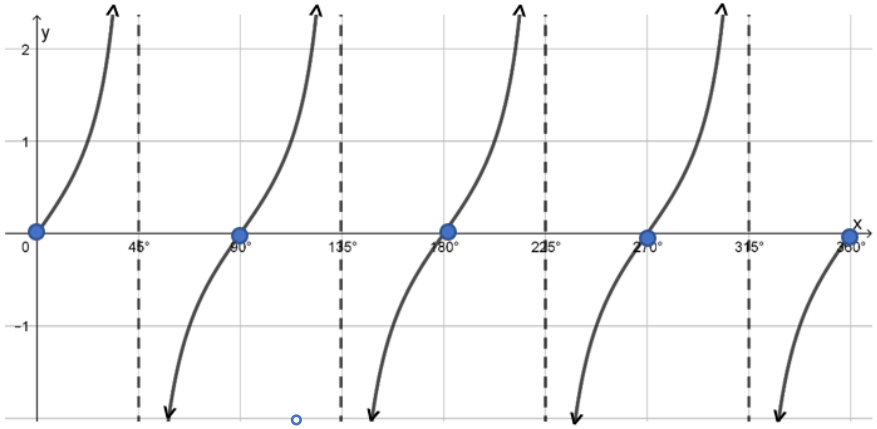
Function	**Value of *k***	**Number of complete cycles**	**Period**	**Calculations**
$y = \sin x$	1	1	360°	Normal period
$y = \sin 3x$		3	120°	$\frac{360}{3} = 120°$
$y = \cos x$	1			Normal period
$y = \cos\frac{1}{2}x$	$\frac{1}{2}$		720°	$\frac{360°}{\frac{1}{2}} = 360° \times 2 = 720°$

Now we can expect the same to happen to the tan graph. Below are the two graphs drawn on different axes.

$y = \tan x$



$y = \tan 2x$



Graph repeats every 90°

From the above, we can conclude that the period of $y = \tan 2x$

$$= \frac{normal}{k} = \frac{180°}{2} = 90°$$

Example 1:

Determine the amplitude and period of the following:

$y = -\cos\frac{3x}{2} + 1$

$a = -1 \ \therefore amplitude = 1$

$$Period = \frac{normal\ period}{k}$$

$$= \frac{360°}{\frac{3}{2}} = 240°$$

Remember:
$y = a\cos kx + q$

Calculator can be used!

Can You?

Write down the amplitude, period range and domain for each of the following functions:

1. $y = 2\sin x$
2. $y = 2\cos 2x + 1$
3. $y = 3\sin\frac{x}{2}$
4. $y = \tan\frac{1}{3}x - 1$

Example 2:

Draw the graph $y = 2\cos 2\theta$ for $\theta \in [-90°; 180°]$

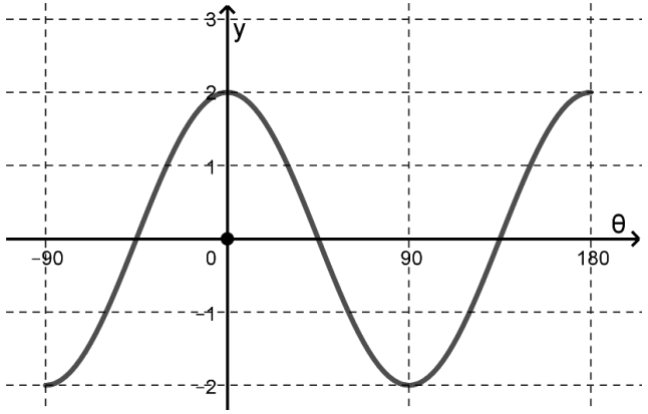
Solution:

Information:

Amplitude = 2

Period: $\frac{360°}{2} = 180°$

as



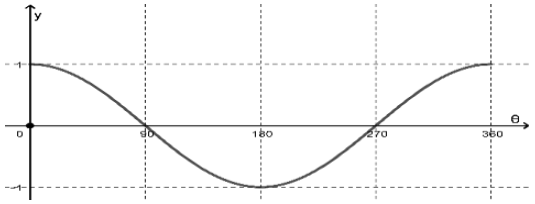
Hints:

1. Draw the rough sketch of the basic graph on the RHS of you page.
2. What changes is needed?

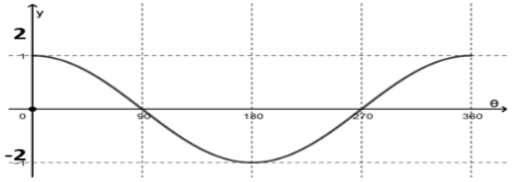
   $a = 2$. Make changes
3. Determine new period
   Add changes to rough drawing
4. Draw final graph for the required domain.

Rough work

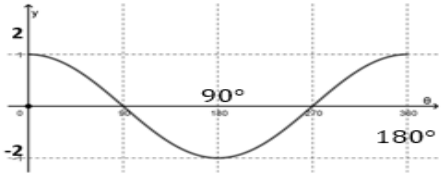
1.



2.



3.



CAN YOU?

Draw neat sketches of the following functions:

1. $f(x) = -\sin\frac{x}{2}$ for $x \in [-90°; 90°]$
2. $g(x) = \cos 2x$ for $x \in [-180°; 180°]$
3. $h(x) = 2\cos 3x$ for $x \in [-120°; 90°]$
4. $k(x) = -\tan 2x - 1$ for $x \in [-90°; 180°]$

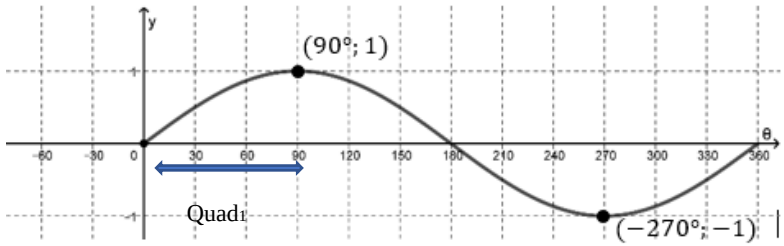
- Always start with the basic function
- Period and Range is important
- Show important facts

The effect of p in $y = a\sin k(x + p) + q$

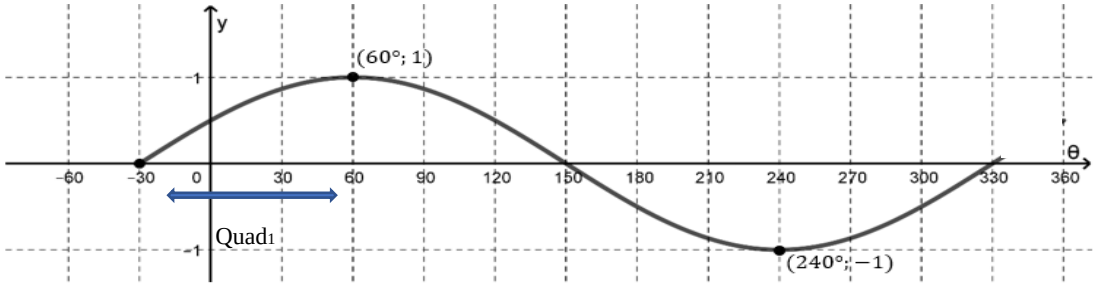
As in Functions, we can expect p to have a similar influence on the trigonometric function. We expect a tranformation to the left or the right. Therfore a **horisontal shift.**

Below are the functions of $\sin x$ and $y = \sin(x + 30°)$:

$y = \sin x$ for $x \in [0°; 360°]$



$y = \sin(x + 30°)$ for $x \in [-30°; 330°]$



Please Note:

- The cycle starts at 0°
- The maximum is at 90°.
- The cycle ends at 360°.
- Compare the quadrants and points

Complete the sentences below:

- cycle starts at ……
- The maximum is at ……
- Is cycle ends at ……..
- We can therefore state that the graph shifted …….. to the left.
- Write down your deductions regarding the quadrants and main points on the graph.

……

……

……

……

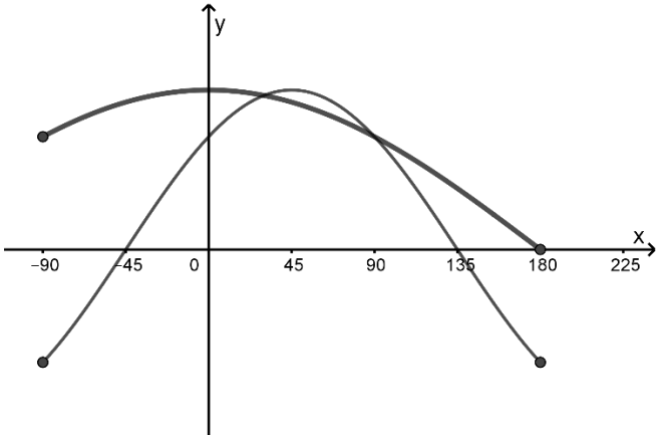
Please take note:

The value of p in the function $y = a \sin k(x + p) + q$

For $(x + p)$ shift to the left
For $(x - p)$ shift to the right

Example 3:

The diagram shows the graphs of $f(x) = \sin(x + p)$ and $g(x) = \cos ax$; $x \in [-90°; °180°]$.



1. Determine the values of p and a.
2. Give the period of g.
3. Determine the range of k, if $k(x) = f(x) - 1$

Solution:

1. $p = 45°$

   $a = \frac{1}{2}$

2. Period: $720°$

3. Range $y \in [-2; 0]; y \in \mathbb{R}$

Hints/ Reasoning:

1. The basic $\sin x$ function starts at 0. The function f starts at $-45°$. The graph moved $45°$ to the left. The fuction must be $f(x) = \sin(x + 45°)$

2. The period of a cos graph is $360°$.
   From $0°$ to $180°$ represents 1 quadrant of the function. 4 quadrants will therfore be $4 \times 180° = 720°$
   To calculate the period we use the formula:

   $$Period = \frac{normal\ period}{a}$$

   $$720° = \frac{360°}{a}$$

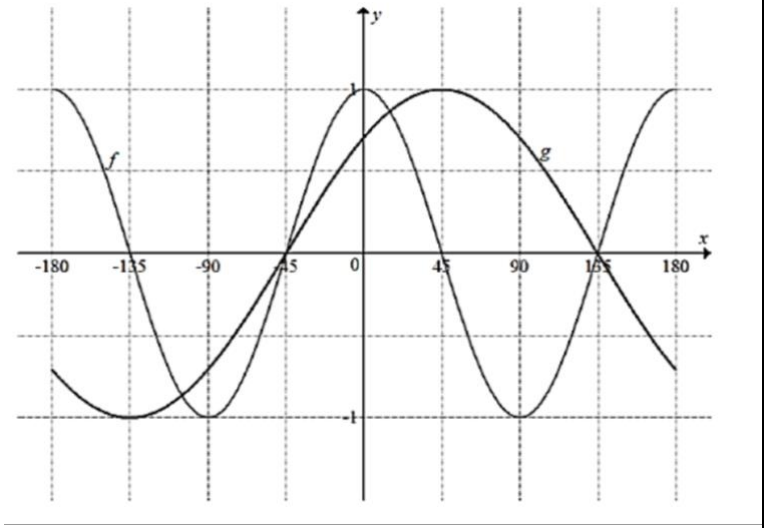
   $$a = \frac{360°}{720°} = \frac{1}{2}$$

3. $f(x) - 1$ means that the graph moves 1 unit downwards.
   The range of f is $y \in [-1; 1]$ for $y \in \mathbb{R}$
   $\therefore$ The range of $k(x) = f(x) - 1$
   The range of $k(x)$: $y \in [-2; 0]$

CAN YOU?

In the diagram below the graphs of $f(x) = a\cos bx$ and $g(x) = \sin(x + p)$ are drawn for $x \in [-180°; 180°]$

1. Write down the values of a, b and p.
2. For which values of x in the given interval does the graph of f increases as the graph of g increases?
3. Write down down the period of $f(x)$
4. Determine the minimum value of h if $h(x) = f(x) - 1$.
5. Describe how the graph of f must be transformed to form graph to form the $k(x) = \sin x$.



Solutions:

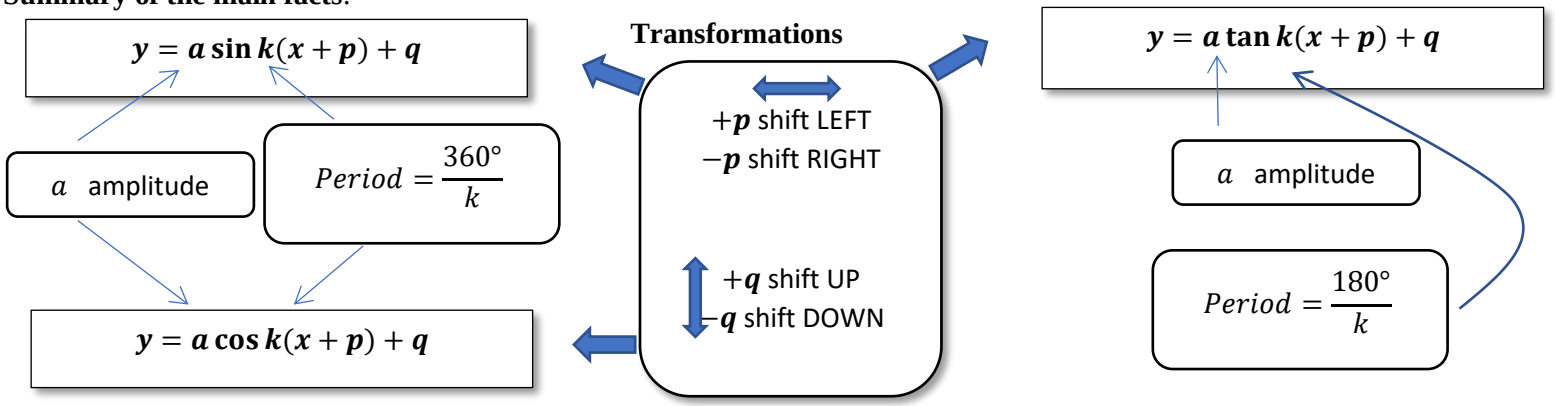
1. $a = 1; b = 2; p = -45°$
2. $x \in [-90°; 0°]$
3. $180°$
4. -2 **[Graph moves down 1 unit]**
5. Move 45° to the right.

ACTIVITIES/ ASSESSMENT

Mind Action Series	Platinum	Clever	Everything Maths (Siyavula)
Ex 2&3: Pg 192-194	Ex 4&5: Pg 124 Ex 11: Pg 135	Ex 6.2-6.6: Pg 189-205	Ex 5.21-5.27 : Pg 202- 221 Ex 5.29-5.31: Pg 227-232

CONSOLIDATION

Summary of the main facts:



Always refer to the basic functions!