SUBJECT and GRADE	Mathematics Gr 11
TERM 3	Week 6
TOPIC	Formulae to solve triangles: Area Rule
AIMS OF LESSON	To introduce learners to formulae to calculate the sizes of angles and lengths of sides of any triangle.
RESOURCES	***Paper based resources*** *Please refer to the section on the Solution of Triangles/ Trigonometry in solving triangles in your Textbook.* ***Digital resources*** *Derivation of the Area formula:* https://www.youtube.com/watch?v=P6H8KIWOFEA
INTRODUCTION	• In the previous lesson we looked at the sin- and cos rule to calculate the unknown sides and/or angles of any triangle. • In this lesson we will look at the area rule that will enable us to calculate the area of any triangle. • This rule again is very important for further study of Trigonometry and you are required to KNOW THE PROOF of it for examination purposes. It forms part of the 12 marks for Bookwork that will be tested in Paper 2.
CONCEPTS AND SKILLS	• Using Theorem of Pythagoras • Solving right-angled triangles • Drawing Perpendicular heights of triangles • Using trig ratios: sin θ, cos θ and tan θ • Using the scientific calculator to do trigonometric calculations

Lesson 1: Revising: Area of a triangle

1. Calculate the area of ΔDEF if $EF = 6$ cm, $\angle F = 90°$ and $DF = 9$ cm

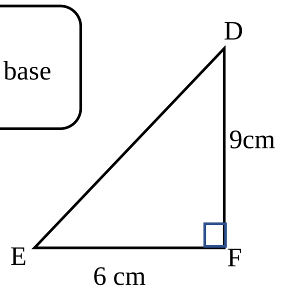
Solution:

⊥ height (DE) is perpendicular to the base (EF)

Area $\Delta DEF = \frac{1}{2}\text{base} \times \perp \text{height}$

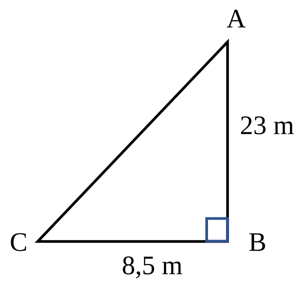
$= \frac{1}{2}(EF)(DF) = \frac{1}{2}6)(9)$

$= 27\ cm^2$



CAN YOU?

1. Calculate the area of ΔABC if $AB = 23$ m, $BC = 8,5$ m and $\angle B = 90°$



Solution: Area $\Delta ABC = 97,75\ m^2$

Here it is clear that we need a ⊥ height (right-angled triangle) to calculate the area. But what if there is NO ⊥ height, thus no right-angled triangle? *[Yes, we can draw one, but what will it's length be?]*

This is why we introduce the Area rule

The Area rule: In ΔABC, we have: Area of $\Delta ABC = \frac{1}{2}ab\sin C$ OR $\frac{1}{2}ac\sin B$ OR $\frac{1}{2}bc\sin A$
Proof : Given any ΔABC. Proof that Area of $\Delta ABC = \frac{1}{2}ab\sin C$ OR $\frac{1}{2}ac\sin B$ OR $\frac{1}{2}bc\sin A$ [We will do this for when $\hat{A}$ is acute and for when $\hat{A}$ is obtuse] (Note: Proof might be different in your textbook)
Construction: Draw ΔABC with h the perpendicular height from B onto AC (fig. 1) or AC produced (fig. 2)

Fig.1

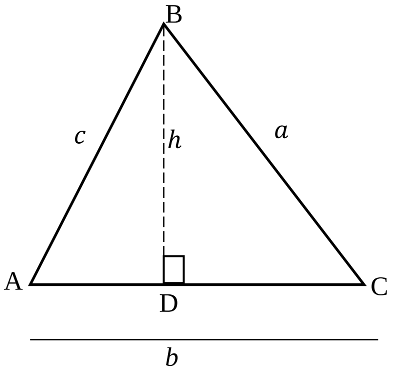
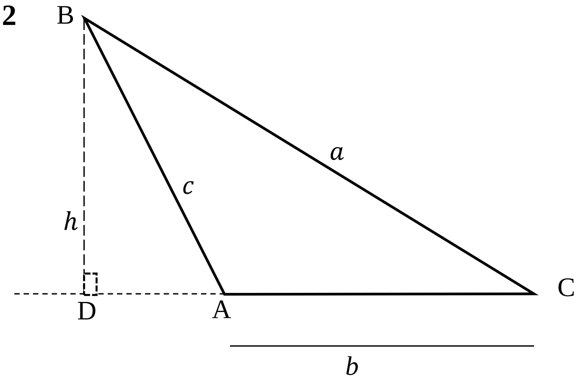


Fig. 2



Area of $\Delta ABC = \frac{1}{2}\text{base} \times \perp \text{height}$

$$= \frac{1}{2}AC \times BD = \frac{1}{2}b \times BD$$

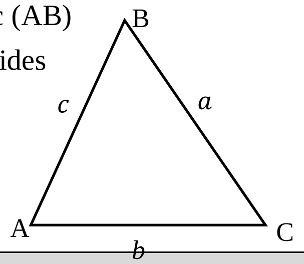
But: In ΔABD $\frac{BD}{c} = \sin A \Rightarrow BD = c\sin A$ OR In ΔBCD $\frac{BD}{a} = \sin C \Rightarrow BD = a\sin C$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}bc\sin A$ OR Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Similarly, by constructing a $\perp$height from C onto AB, we can proof that: Area of $\Delta ABC = \frac{1}{2}ac\sin B$

Note:

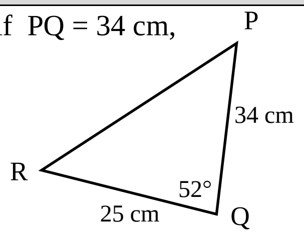
- In the Area rule: Area of $\Delta ABC = \frac{1}{2}bc\sin A$, the sides b (AC) and c (AB) are adjacent to the angle A; we say $\hat{A}$ is the **included angle** to the 2 sides
- So, to calcjulate the area, we **need 2 sides** and the **included angle**
- Area is measured in **(units)**^{2}



Lesson 2/3: Applying the Area-rule

Examples:

1. Calculate the Area of ΔPQR if PQ = 34 cm, QR = 25 cm and $P\hat{Q}R = 52°$



Solution:
[2 sides and included angle given]

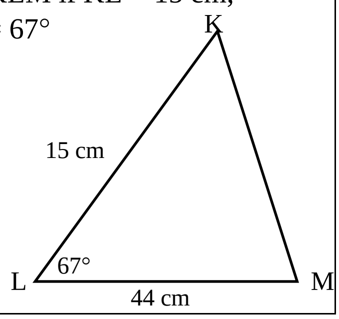
Area of $\Delta PQR = \frac{1}{2}pr\sin Q$

$= \frac{1}{2}(25)(34)\sin 52°$ — PQ = r and QR = p

= 334,9 cm^2

Can you do?

1. Calculate the Area of ΔKLM if KL = 15 cm, LM = 44 cm and $K\hat{L}M = 67°$



Solution: Area ΔKLM = 303,77 cm^2

2. Calculate the Area of ΔABC if $a = 7$ m, $b = 11$ cm and $\hat{B} = 98°$

Solution: [2 sides and **not an included angle** given]

To calculate the included angle C, we 1st use the sin-rule to calculate $\hat{A}$ (since 1 of the given sides is opposite $\hat{A}$)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

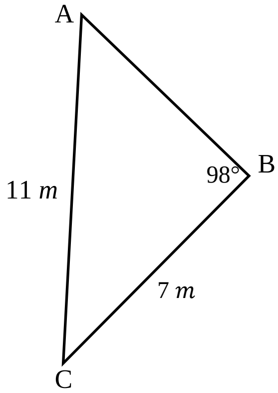
$$\therefore \frac{\sin A}{7} = \frac{\sin 98°}{11}$$

$$\therefore \sin A = \frac{7 \sin 98°}{11}$$

$$= 0{,}63017\ldots$$

$\therefore \hat{A} = 39{,}06°$

Hence: $\hat{C} = 180° - 39{,}06° - 98°$
$= 42{,}94°$

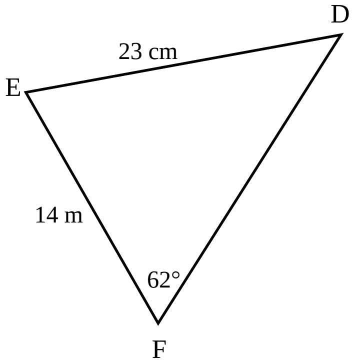


Now: Area of $\Delta ABC = \frac{1}{2} ab \sin C = \frac{1}{2}(7)(11) \sin 42{,}94°$

$26{,}23\ m^2$

Can you do?

2. Calculate the Area of ΔDEF if $d = 14$ cm, $f = 23$ cm and $\hat{F} = 62°$



Solution: Area of $\Delta DEF = 160{,}5\ cm^2$ [$\hat{E} = 85{,}49°$]

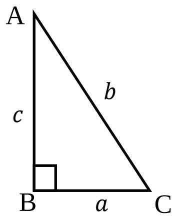
Note: If we have a right-angled Δ, applying the Area rule just leads to the normal Area formule

From the diagram:

Area of $\Delta ABC = \frac{1}{2} ac \sin B$

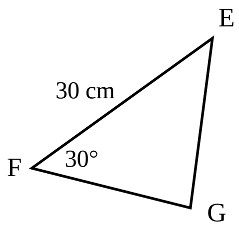
$= \frac{1}{2} ac \sin 90°$ (sin 90°=1)

$= \frac{1}{2} ac(1) = \frac{1}{2} AB \times BC = \frac{1}{2}$ base $\times \perp$ height



3. The area of $\Delta FGH = 100\ cm^2$, $\hat{F} = 30°$ and $g = 30$ cm. Determine the length of h

Solution:



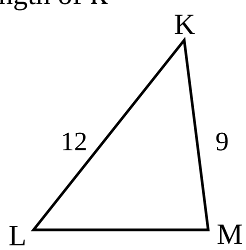
Area of $\Delta FGH = \frac{1}{2} gh \sin F = 100$

$\therefore \therefore \frac{1}{2}(30)h \sin 30° = 100$

$\therefore h = \frac{100}{15 \sin 30°} = 13{,}3$ cm

Can you do?

3. The area of $\Delta KLM = 78{,}98\ cm^2$, $KL = 12$ cm and $KM = 9$ cm. Calculate the length of $\hat{K}$



Solution: $\hat{K} = 46°$

Lesson 4: Mixed problems on sin/cos/area rule

Note:

- Sin-rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ OR $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

  ✓ Used when **2 sides and an angle,** *opposite* 1 of the 2 sides, are given or when **2 angles and a side** are given

- Cos-rule: $a^2 = b^2 + c^2 - 2bc\cos A$ to calculate a side OR

  $\cos A = \frac{b^2+c^2-a^2}{2bc}$ for calculating an angle.

  ✓ Used when **2 sides and** the **included angle,** or when **3 sides** are given

- Area rule: Area of $\Delta ABC = \frac{1}{2}ab\sin C$ OR $\frac{1}{2}ac\sin B$ OR $\frac{1}{2}bc\sin A$

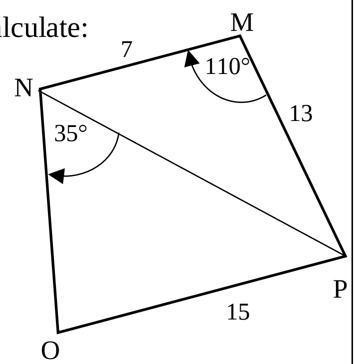
  ✓ Used when **2 sides and** the **included angle** are given.

- Usually cos-rule are used once in a triangle, thereafter use the sin-rule.
- Area rule used only when area are given or asked.

Example 1.

Use the diagram and calculate:

(a) Area of ΔMNP
(b) Length of NP
(c) $N\hat{O}P$



Solutions:

(a) Area of $\Delta MNP = \frac{1}{2}np\sin M$

$= \frac{1}{2}(13)(7)\sin 110° = 42,76$ (units)^2

2 sides and included angle given

(b) In ΔMNP: $m^2 = n^2 + p^2 - 2np\cos M$

$\therefore NP^2 = (13)^2 + (7)^2 - 2(13)(7)\cos 110°$

$\therefore NP^2 = 280,247\ldots$

$\therefore NP = 16,74$ units

2 sides and an angle opposite 1 of the sides given

(c) In ΔNOP: $\frac{\sin O}{o} = \frac{\sin N}{n}$

$\therefore \frac{\sin O}{16,74} = \frac{\sin 35°}{15} \Rightarrow \sin O = \frac{16,74\sin 35°}{15}$

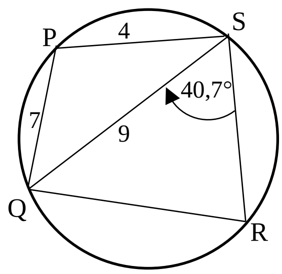
$= 0,64$

$\therefore \hat{O} = 39,8°$

Can you do?

Refer to the diagram and calculate:

(a) $\hat{P}$ (obtuse)
(b) Area of ΔPQS
(c) $\hat{R}$, with reasons
(d) SR



Solution: QR = 14,05

ACTIVITIES/ ASSESSMENT	*Please do the exercises as they appear in you textbook at the end of the **area rule***

CONSOLIDATION:	• Sin-rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ OR $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ✓ Used when **2 sides and an angle,** *opposite* 1 of the 2 sides, are given or when **2 angles and a side** are given • Cos-rule: $a^2 = b^2 + c^2 - 2bc \cos A$ to calculate a side OR $\cos A = \frac{b^2+c^2-a^2}{2bc}$ for calculating an angle. ✓ Used when **2 sides and** the **included angle** are given, or **3 sides** are given. • Area rule: Area of $\Delta ABC = \frac{1}{2}ab \sin C$ OR $\frac{1}{2}ac \sin B$ OR $\frac{1}{2}bc \sin A$ ✓ Used when **2 sides and** the **included angle** are given • Area rule used only when area are given or asked. • The cos-rule are used once in a triangle and thereafter use sin-rule to calculate other angles/ sides. • In the next lesson we will look at the application of the rules on Problems in 2D. • Thank you for participating in this lesson and please continue to work through the lessons as they are made available. • Remember: Your hard work will reap success at the end!! KEEP IT UP!!