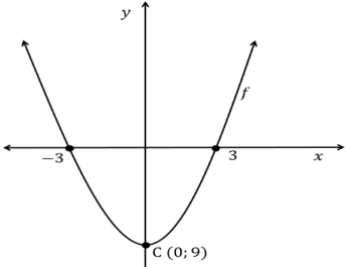
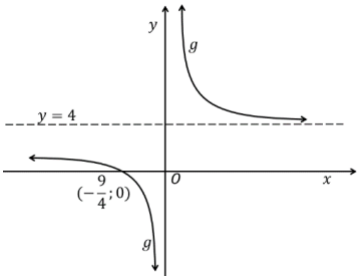
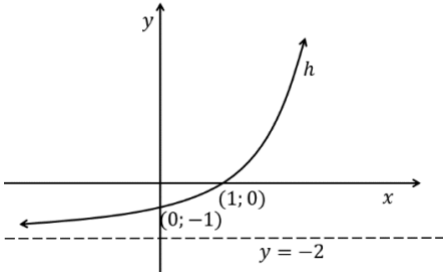





| SUBJECT and GRADE | MATHEMATICS GR 11 | | | |
|--------------------------------|---|---|---|---|
| TERM 3 | Week 2 | | | |
| TOPIC | FUNCTIONS | | | |
| AIMS OF LESSON | <ul style="list-style-type: none"> • Sketching the graphs of a parabolic-, hyperbolic- and exponential function. • Introducing the Parabola equations in turning point form and standard form. • Use concepts of shape, turning point, axes of symmetry, intercepts and asymptotes as they relate to the graphs. | | | |
| RESOURCES | <i>Paper based resources</i> | <i>Digital resources</i> | | |
| | Please go to the Functions section in your Mathematics Textbook. | Parabola: https://bit.ly/2KMIuwW ; https://bit.ly/2ycgLTF ; https://bit.ly/3eWbfFk Hyperbola: https://bit.ly/3aQokg4 ; https://bit.ly/3aOhEiz ; https://bit.ly/3bSa8Va Exponential: https://bit.ly/2KLNbxg | | |
| INTRODUCTION | <p>In Grade 10 you have learnt to sketch graphs of the form: $y = ax^2 + q$, $y = \frac{a}{x} + q$ and $y = a \cdot b^x +$</p> <p>Here are some examples of graphs sketched in GR10.</p> |  |  |  |
| • Standard form | $y = ax^2 + q$ | $y = \frac{a}{x} + q$ | $y = a \cdot b^x + q$ | |
| • Equation for this example | $f(x) = x^2 - 9$ | $g(x) = \frac{9}{x} + 4$ | $h(x) = 2^x - 2$ | |
| • x –intercepts (y = 0) | $(-3; 0)$ and $(3; 0)$ | $(-\frac{9}{4}; 0)$ | $(1; 0)$ | |
| • y –intercept (x = 0) | $(0; 9)$ | none | $(0; -1)$ | |
| • Asymptotes | | horizontal asymptote $y = 4$ vertical asymptote $x = 0$ | horizontal asymptote $y = -2$ | |
| • Other | $a > 0 \rightarrow$ arms up  | $k > 0 \rightarrow$ graph in 1st and 3rd quadrant | $b > 1 \rightarrow$ graph is increasing | |



CONCEPTS AND SKILLS

(1) PARABOLA

In this lesson we are going onto sketching the same graphs, but we will include an extra parameter p .

$$y = a(x + p)^2 + q, y = \frac{k}{x+p} + q \text{ and } y = a \cdot b^{x+p} + q$$

We will discover that this variable will translate (move) the graphs left or right.

HOW TO SKETCH THE GRAPH: $y = a(x + p)^2 + q$

There are **THREE** concepts that the examiner will award marks for:

- Turning point
- x –intercepts
- y – intercept

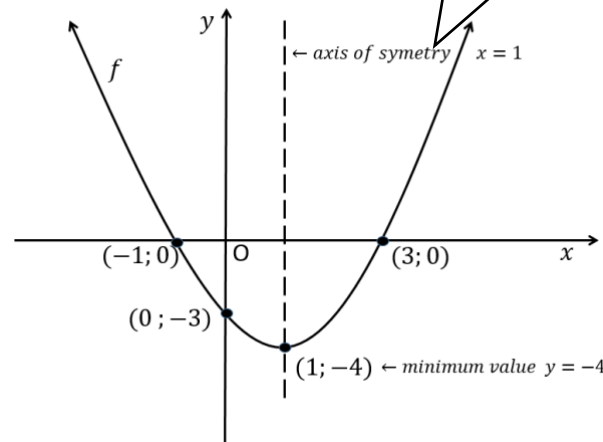
EXAMPLE 1

1.1 Draw the sketch $f(x) = (x - 1)^2 - 4$

Clearly show all intercepts with the axes and the coordinates of the turning point.

ANSWER:

- $a > 0 \rightarrow$ arms up
- **TP** $(1; -4)$
- **x –intercepts ($y = 0$)**
 $0 = (x - 1)^2 - 4$
 $0 = x^2 - 2x + 1 - 4$
 $0 = x^2 - 2x - 3$
 $0 = (x - 3)(x + 1)$
 $\therefore x = 3 ; x = -1$
- **y –intercept ($x = 0$)**
 $y = (0 - 1)^2 - 4$
 $y = -3$



It is always a good idea to indicate the axis of symmetry with a broken line

1.2 Does this graph have a minimum or a maximum value? Give the value.

ANSWER:

Minimum value of y is -4

NOTE:

$f(x)$ –notation:

To distinguish between different graphs, we can name them with letters of the alphabet. For this reason, we use $f(x)$, $g(x)$, $h(x)$ etc. in the place of the y –value.

CAN YOU?

- 1) Draw the sketch $g(x) = (x + 2)^2 - 9$
Clearly show all intercepts with the axes and the coordinates of the turning point.
- 2) Does this graph have a minimum or a maximum value? Give the value.
- 3) Give the equation for the axis of symmetry.

ANSWER:

1)

x –intercepts ($y = 0$)

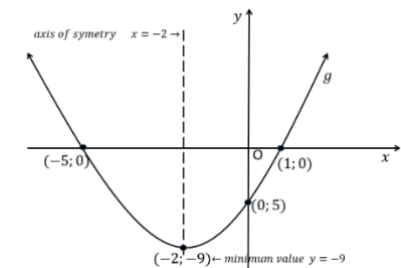
$$0 = (x + 2)^2 - 9$$

$$0 = x^2 + 4x + 4 - 9$$

$$0 = x^2 + 4x - 5$$

$$0 = (x + 5)(x - 1)$$

$$x = -5 ; x = 1$$



- 2) minimum value of y is -9
- 3) $x = -2$



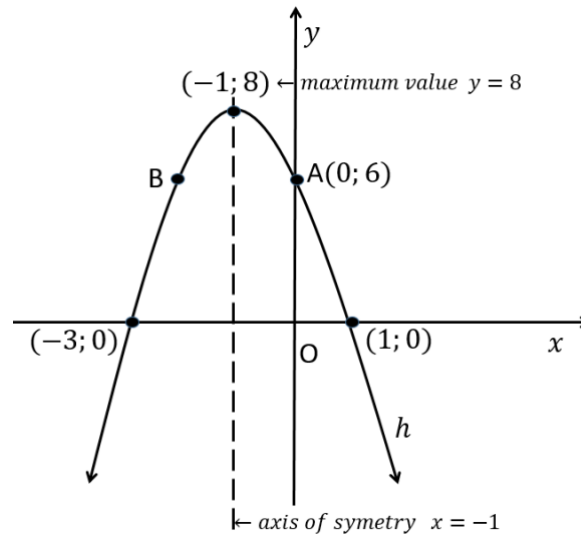
EXAMPLE 2

2.1 Draw the sketch $h(x) = -2(x + 1)^2 + 8$

Clearly show all intercepts with the axes and the turning point.

ANSWER:

- $a < 0 \rightarrow$ arms down
- **TP $(-1; 8)$**
- **x -intercept $(y = 0)$**
 $0 = -2(x + 1)^2 + 8$
 $0 = -2(x^2 + 2x + 1) + 8$
 $0 = -2x^2 - 4x - 2 + 8$
 $0 = -2x^2 - 4x + 6$
 $0 = -2(x^2 + 2x - 3)$
 $0 = -2(x + 3)(x - 1)$
 $\therefore x = -3 ; x = 1$
- **y -intercept $(x = 0)$**
 $y = -2(0 + 1)^2 + 8$
 $y = 6$



Symmetry in sketch:

2.2 Give the coordinates of B, if B is the reflection of A(0; 6) around the axes of symmetry.

ANSWER:

B(-2; 6)

NOTE:

The -2 will NOT influence the coordinates of the turning point

CAN YOU?

4) Draw the sketch $k(x) = 2(x - 2)^2 - 8$

Clearly show all intercepts with the axes and the coordinates of the turning point.

5) Give the coordinates of C, if C is the reflections of D(1; -6) around the axes of symmetry.

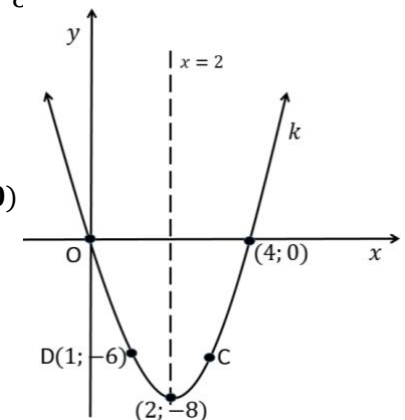
ANSWER:

4)

- $a > 0 \rightarrow$ arms up
- **TP $(2; -8)$**
- **x -intercept $(y = 0)$**
 $0 = 2(x - 2)^2 - 8$
 $0 = 2(x^2 - 4x + 4) - 8$
 $0 = 2x^2 - 8x + 8 - 8$
 $0 = 2x^2 - 8x$
 $0 = 2x(x - 4)$
 $\therefore x = 0 ; x = 4$

- **y -intercept $(x = 0)$**
 $y = 2(0 - 2)^2 - 8$
 $y = 0$

5) C(3; -6)





EXAMPLE 3

3.1 Draw the sketch $f(x) = -2x^2 - 4x + 6$

Clearly show all intercepts with the axis and the turning point.

ANSWER:

- $a < 0 \rightarrow$ arms down
- **TP**

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-4)}{2(-2)}$$

$$x = -1$$

Substitute this value into the **ORIGINAL EQUATION** to calculate the **y** –coordinate.

$$f(-1) = -2(-1)^2 - 4(-1) + 6$$

$$f(-1) = 8$$

Turning Point $(-1; 8)$

- **x** –intercepts ($y = 0$)

$$0 = -2x^2 - 4x + 6$$

$$0 = -2(x^2 + 2x - 3)$$

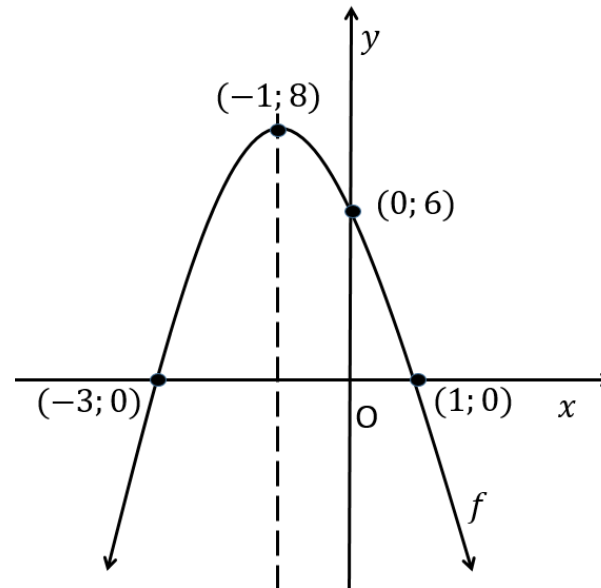
$$0 = -2(x + 3)(x - 1)$$

$$\therefore x = -3 ; x = 1$$

- **y** –intercept ($x = 0$)

$$y = -2(0)^2 - 4(0) + 6$$

$$y = 6$$



NOTE:
This equation is in standard form.
We cannot “see” the turning point in this form.

We must use the following formula to get the symmetry axis:

$$x = \frac{-b}{2a}$$

CAN YOU?

6) Draw the sketch $g(x) = x^2 - 8x + 16$

Clearly show all intercepts with the axis and the coordinates of the turning point

ANSWER:

- $a > 0 \rightarrow$ arms up
- **TP**

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)}$$

$$x = 4$$

$$f(4) = (4)^2 - 8(4) + 16$$

$$f(4) = 0$$

TP $(4; 0)$

- **x** –intercepts ($y = 0$)

$$0 = x^2 - 8x + 16$$

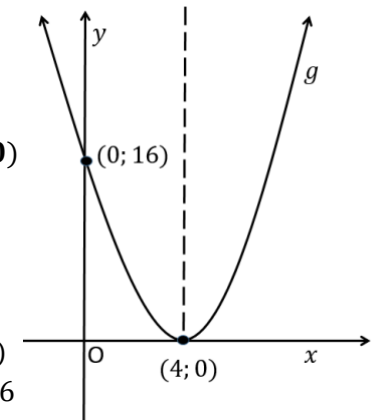
$$0 = (x - 4)(x - 4)$$

$$\therefore x = 4 ; x = 4$$

- **y** –intercept ($x = 0$)

$$y = (0)^2 - 8(0) + 16$$

$$y = 16$$





CONCEPTS AND SKILLS

(2) HYPERBOLA

HOW TO SKETCH THE GRAPH: $y = \frac{a}{x+p} + q$

There are **THREE** concepts that the examiner will award marks for:

- Horizontal and vertical asymptotes.
- x –intercept
- y – intercept

EXAMPLE 4

4.1) Sketch the graph of $h(x) = \frac{8}{x+1} - 4$ clearly show all asymptotes and the intercepts with the axes.

ANSWER:

- Horizontal asymptote: $y = -4$ and vertical asymptotes: $x = -1$

- x –intercept ($y = 0$)

$$0 = \frac{8}{x+1} - 4$$

$$4 = \frac{8}{x+1}$$

$$4(x+1) = 8$$

$$4x + 4 = 8$$

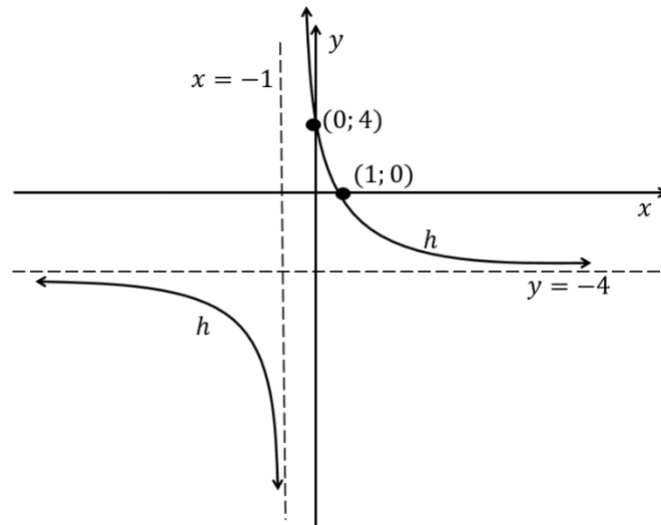
$$4x = 4$$

$$x = 1$$

- y – intercept

$$y = \frac{8}{0+1} - 4$$

$$y = 4$$



CAN YOU?

7) Sketch the graph of $p(x) = \frac{-4}{x-1} - 2$

clearly show all asymptotes and the intercepts with the axes.

ANSWER:

- Horizontal asymptote: $y = -2$

- vertical asymptote: $x = 1$

- x –intercept ($y = 0$)

$$0 = \frac{-4}{x-1} - 2$$

$$2 = \frac{-4}{x-1}$$

$$2(x-1) = -4$$

$$2x - 2 = -4$$

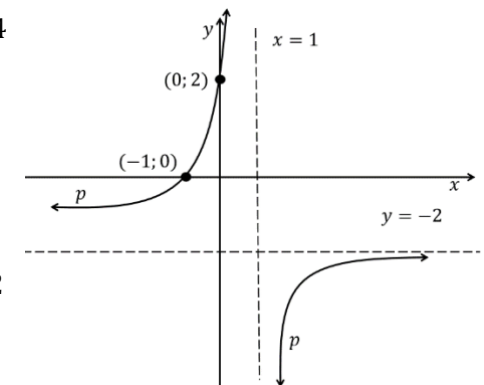
$$2x = -2$$

$$x = -1$$

- y – intercept

$$y = \frac{-4}{0-1} - 2$$

$$y = 2$$





CONCEPTS AND SKILLS

(2) EXPONENTIAL GRAPH

HOW TO SKETCH THE GRAPH: $y = a \cdot b^{x+p} + q$

There are **THREE** concepts that the examiner will award marks for:

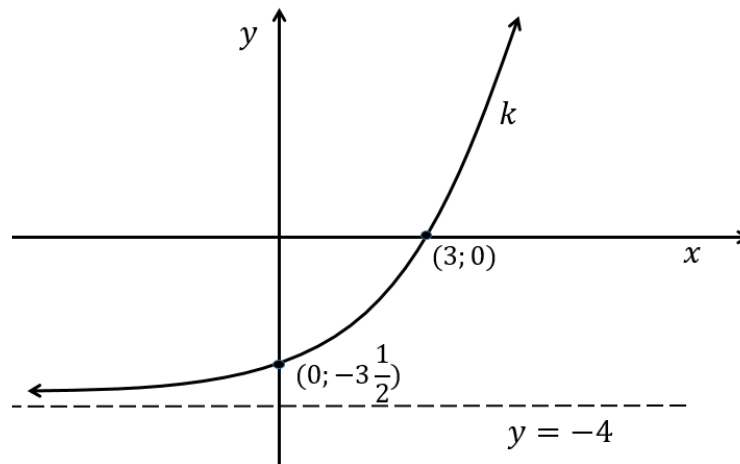
- Horizontal asymptote.
- x –intercept
- y – intercept

EXAMPLE 5

5.1 Sketch the graph of $k(x) = 2^{x-1} - 4$ clearly show all asymptote(s) and the intercepts with the axes.

ANSWER:

- Horizontal asymptote: $y = -4$
- x –intercept ($y = 0$)
 $0 = 2^{x-1} - 4$
 $4 = 2^{x-1}$
 $2^2 = 2^{x-1}$
 $2 = x - 1$
 $3 = x$
- y – intercept
 $y = 2^{0-1} - 4$
 $y = \frac{1}{2} - 4$
 $y = -3\frac{1}{2}$

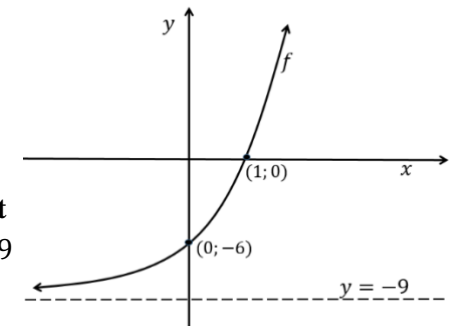


CAN YOU?

8) Sketch the graph of $f(x) = 3^{x+1} - 9$ clearly show all asymptotes and the intercepts with the axes.

ANSWER:

- Horizontal asymptote: $y = -9$
- x –intercept ($y = 0$)
 $0 = 3^{x+1} - 9$
 $9 = 3^{x+1}$
 $3^2 = 3^{x+1}$
 $2 = x + 1$
 $1 = x$
- y – intercept
 $y = 3^{0+1} - 9$
 $y = 3 - 9$
 $y = -6$



9) Give the equation for g if g is the graph after f has moved 6 units up and 1 unit to the left.

ANSWER:

$g(x) = 3^{x+2} - 3$



HOW TO SKETCH THE GRAPH: $y = a \cdot b^{x+p} + q$

EXAMPLE 6

6.1) Sketch the graph of $k(x) = \frac{1}{2}^{x-2} - 8$ clearly show all asymptote(s) and the intercepts with the axes.

ANSWER:

- **Horizontal asymptote:** $y = -8$

- **x-intercept ($y = 0$)**

$$0 = \frac{1}{2}^{x-2} - 8$$

$$8 = (2^{-1})^{x-2}$$

$$2^3 = 2^{-x+2}$$

$$3 = -x + 2$$

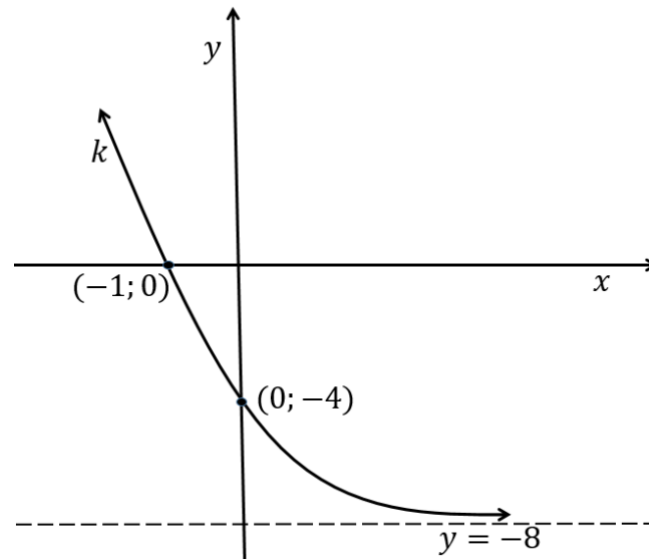
$$x = -1$$

- **y-intercept**

$$y = \frac{1}{2}^{0-2} - 8$$

$$y = 4 - 8$$

$$y = -4$$



CAN YOU?

10) Sketch the graph of $g(x) = \frac{1}{5}^{x+1} - 1$ clearly show all asymptotes and the intercepts with the axes.

ANSWER:

- **Horizontal asymptote:** $y = -1$

- **x-intercept ($y = 0$)**

$$0 = \frac{1}{5}^{x+1} - 1$$

$$1 = \frac{1}{5}^{x+1}$$

$$5^0 = (5^{-1})^{x+1}$$

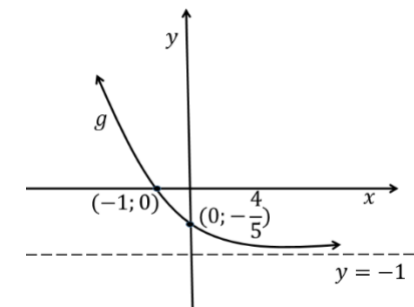
$$0 = -x - 1$$

$$x = -1$$

- **y-intercept**

$$y = \frac{1}{5}^{0+1} - 1$$

$$y = -\frac{4}{5}$$



11) Give the equation for h if h is the graph after f has moved 1 units up and 1 unit to the right.

ANSWER:

$$h(x) = \frac{1}{5}^x$$

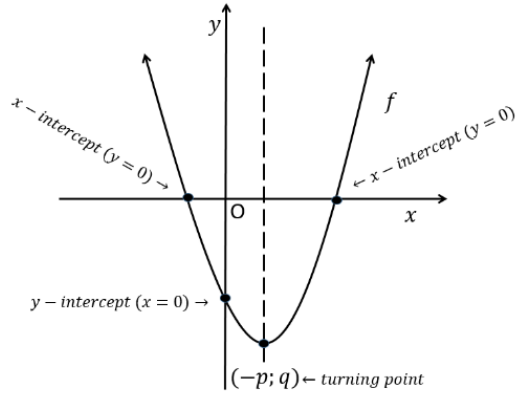
ACTIVITIES/ASSESSMENT

- VIA AFRICA p 52 – 56 Question 1 – 9
- MIND ACTION SERIES Exercise 1 – 6 p 87 – 114
- SIYAVULA Exercise 5 – 4 p154 ; Exercise 5 – 5 p155 and Exercise 5 – 6 p161
- PLATINUM Exercise 3 p 86 ; Exercise 4 p 89 en Exercise 7 p 97



CONSOLIDATION

PARABOLA $y = a(x + p)^2 + q$

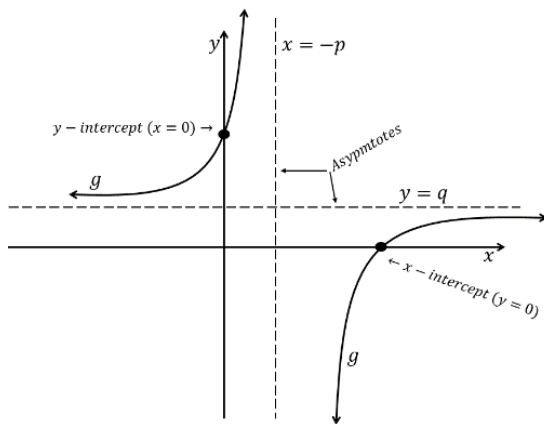


- x - intercept ($y = 0$) & y - intercept ($x = 0$)
- Parabola: Turning point $(-p; q)$

| | $p > 0$ | | $p < 0$ | |
|---------|---------|---------|---------|---------|
| | $a < 0$ | $a > 0$ | $a < 0$ | $a > 0$ |
| $q > 0$ | | | | |
| $q < 0$ | | | | |

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HYPERBOLA $y = \frac{a}{x+p} + q$



- x - intercept ($y = 0$) & y - intercept ($x = 0$)
- Asymptotes $x = -p$ and $y = q$

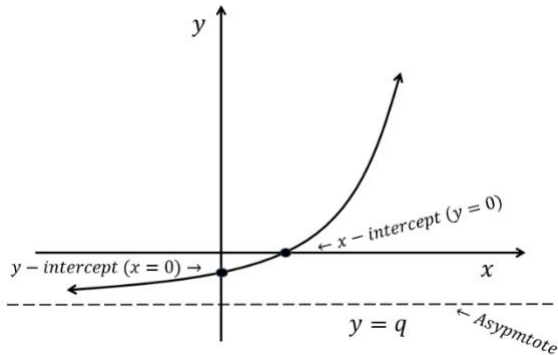
| | $p > 0$ | | $p < 0$ | |
|---------|---------|---------|---------|---------|
| | $a < 0$ | $a > 0$ | $a < 0$ | $a > 0$ |
| $q > 0$ | | | | |
| $q < 0$ | | | | |

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ESPOLENTIAL GRAPH:

$y = a \cdot b^{x+p} + q$



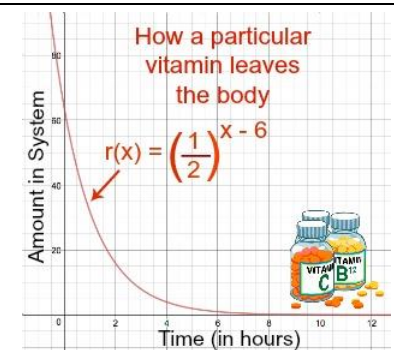
- $x - intercept (y = 0)$ & $y - intercept (x = 0)$
- Asymptote $y = q$

| | | | | | |
|---------|---------|---------|-------------|---------|---------|
| $b > 1$ | $a < 0$ | $a > 0$ | $0 < b < 1$ | $a < 0$ | $a > 0$ |
| $q > 0$ | | | $q > 0$ | | |
| $q < 0$ | | | $q < 0$ | | |

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VALUES:

Concepts/skills in real life scenarios



<http://passyworldofmathematics.com/sydney-harbour-bridge-mathematics/>

<https://study.com/academy/lesson/modeling-the-real-world-with-families-of-functions.html>