SUBJECT and GRADE	MATHEMATICS GRADE 11	
TERM 4	WEEK 1	
TOPIC	PROBABILITY	
AIMS OF LESSON	1. Revise the • **ADDITION RULE:** $\mathbf{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$ • ADDITION RULE FOR MUTUALLY EXCLUSIVE EVENTS: $P(A \text{ or } B) = P(A) + P(B)$ • **COMPLEMENTARY RULE:** $\mathbf{P(\text{not } A) = 1 - P(A)}$ 2. The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Probability section in your Mathematics Textbook.	*Calculating Probability Using Venn Diagrams* https://www.youtube.com/watch?v=ErJ2F8lWJKc *Grade 12 Probability Past Exam Question 6 Nov 2013 Part 1| NTE* https://www.youtube.com/watch?v=G1zP_7dqOVg *Determine Probability from a Venn Diagram (Basis, And, Or, Complement)* https://www.youtube.com/watch?v=IPlPTu-L0BI *Shading Venn Diagram Regions* https://www.youtube.com/watch?v=059SNMvGEzE

INTRODUCTION: PRE-KNOWLEDGE

- **CAPS – GR 10**

a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome.

(b) Venn diagrams as an aid to solving probability problems.

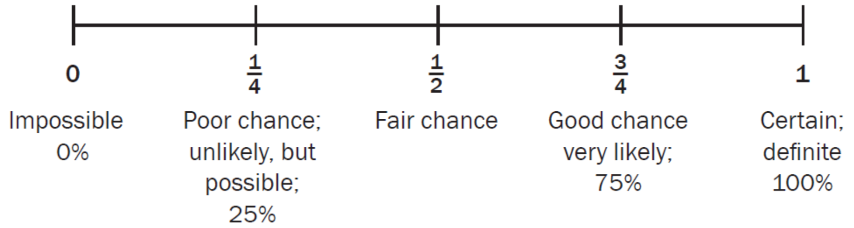
(c) Mutually exclusive events and complementary events.

(d) The identity for any two events A and B: P(A or B) = P(A) + (B) − P(A and B)

What learners should already know from previous grades:

- Probability is the likelihood of something happening in the future. It is expressed as a number between **zero** (can never happen) to **1** (will always happen). It can be expressed as a fraction, a decimal or as a percentage. So, a probability of 5 out of 8 can be written as $\frac{5}{8}$ or as 0,625 or as 62,5%.

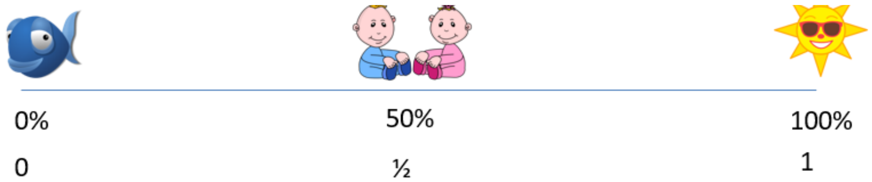
- We can use a probability scale to decide what chance there is of an event happening.



The probability that a fish will be alive after 2 days out of the water is 0%

The probability that the pregnant lady will have a girl is 50%.

The probability that the sun will come up tomorrow is 100%.



- The formula we use to calculate probability is:

The **probability** of an event = $\frac{\text{Number or favorable outcomes}}{\text{Total number or possible outcomes}}$

$$P(A) = \frac{n(E)}{n(S)}$$

• **TERMINOLOGY**	• An **EVENT** is a happening or an activity that has outcomes or results. • An **OUTCOME** is the possible result of an event. • The **SAMPLE SPACE** is the set of all possible outcomes.	**Example:** *Rolling an even number is an event with given outcomes.* **Example:** The possible outcomes: • *of rolling a dice is 1, 2, 3, 4, 5 and 6* • *If you flip a coin: is H (heads) or T (tails).*
• **SYMBOLS AND SETS USED IN PROBABILITY**	S	Sample Space.
	P(A)	probability that an element from set A will occur.
	n(A)	the number of elements in set A.
	A'	all the elements of the sample space that are NOT in set A. (Complement of A)
	A ∪ B A or B.	It means the union of the two sets and represents the total of all the elements that are in set A or set B.
	A ∩ B A and B	It means the intersection of sets A and B and represents all the elements that they share.
	P(A ∩ B)	the probability that an element from (A ∩ B) will occur.
	n(A ∪ B)	the number of elements in set A or set B.

• **PROBABILITY RULES**	❖ Event A and B is **exhaustive**	❖ $\mathbf{P(A\ or\ B) = 1}$
	❖ Events A and B are **mutually exclusive**	❖ $\mathbf{P(A\ and\ B) = 0}$
	❖ A and B are **complimentary events** → (mutually exclusive and exhaustive)	❖ $\mathbf{P(A) + P(B) = 1}$ ❖ $\mathbf{P(A) + P(not\ A) = 1}$ ❖ $\mathbf{P(A') = 1 - P(A)}$
	❖ Addition Rule	❖ $\mathbf{P\ (A\ or\ B)\ =\ P(A) + P(B) - P\ (A\ and\ B)}$ **or** ❖ $\mathbf{P(A \cup B)\ = P(A) + P(B) - P\ (A\ \cap B)}$

EXAMPLE 1:

NOTE:

There are 52 cards in a pack of playing cards.

There are 13 cards in a suit:
2 – 10, J, Q, K & A

There are 4 suits:

- 13 diamonds ♦
- 13 spades ♠
- 13 hearts ♥
- 13 clubs ♣

1.1 The probability that a you will pick a Jack of diamond out of a pack of cards:

Number of favourable outcomes

$$P(J\blacklozenge) = \frac{1}{52}$$

Total number of outcomes

1.2 The probability that a you will pick diamond out of a pack of cards:

$$P(\blacklozenge) = \frac{13}{52} = \frac{1}{4}$$

Always simplify!

1.3 The probability that a you will pick a 17 out of a pack of cards:

$$P(17) = \frac{0}{52} = 0$$

CAN YOU?

QUESTION 1

1.1 Determine the probability that a you will throw a four with a dice.

1.2 Determine the probability that a you will throw an even number with a dice.

1.3 Determine the probability that a you will throw a seven with a dice:

SOLUTION

1.1 $\text{P}(4) = \frac{1}{6}$

1.2 $\text{P(even)} = \frac{3}{6} = \frac{1}{2}$

1.3 $\text{P}(7) = \frac{0}{6} = 0$

ADDITION RULE

$$\mathbf{P\ (A\ or\ B)\ =\ P(A) + P(B) - P\ (A\ and\ B)}$$

or

$$\mathbf{P(A \cup B)\ =\ P(A) + P(B) - P\ (A\ \cap B)}$$

EXAMPLE 2:

Determine the probability that you will get a ten or a heart when you draw a card from a pack of playing cards.

$$\text{P}(10 \text{ or } ♥) = \text{P}(10) + \text{P}(♥) - \text{P}(10 \text{ en } ♥)$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52} = \frac{4}{13}$$

CAN YOU? QUESTION 2

Determine the probability that you will get a six or a three when throwing a dice.

SOLUTION

$$\text{P}(6 \text{ or } 3) = \text{P}(6) + \text{P}(3) - \text{P}(6 \text{ and } 3)$$
$$= \frac{1}{6} + \frac{1}{6} - 0$$
$$= \frac{2}{6} = \frac{1}{3}$$

EXAMPLE 3:

During an experiment it was found that P(A) = 0,25 ; P(B) = 0,5 and P(A or B) = 0,625. Determine P (A and B)

$$\text{P (A or B)}\ =\ \text{P(A)} + \text{P(B)} - \text{P (A and B)}$$
$$0{,}625\ =\ 0{,}25 + 0{,}5 - x$$
$$\therefore x = 0{,}125$$

CAN YOU? QUESTION 3

Given P (A or B) = 0,6; P(B) = 0,4 and P(A and B) = 0,3.

Determine:

3.1 P(A)

3.2 If the events A and B are mutually exclusive.
Give a reason for your answer.

SOLUTION

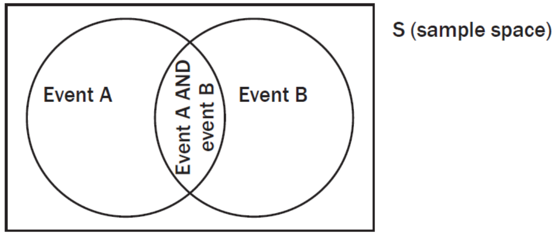
3.1 $\text{P (A or B)}\ =\ \text{P(A)} + \text{P(B)} - \text{P (A and B)}$

$0{,}6\ =\ x + 0{,}4 - 0{,}3$

$\therefore x = 0{,}5$

3.2 A and B are NOT mutually exclusive. P (A and B) ≠ 0

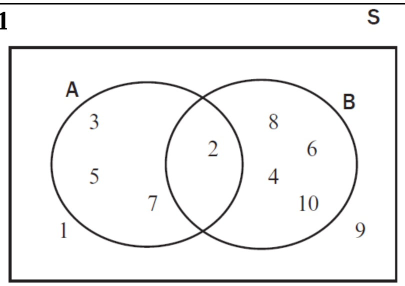
- **VENN-DIAGRAMS**
- We use Venn diagrams to help us to represent different events.
- Venn Diagram consists of circles and a rectangle.
- The rectangle S represents the sample space (all the possible outcomes).
- Each circle inside S represents a different event.
- If the two circles intersect, the intersection shows which outcomes belong to both events.



EXAMPLE 4:

The purpose of these examples is to revise some terminology: *Inclusive *Exhaustive *Mutually exclusive *Complementary events

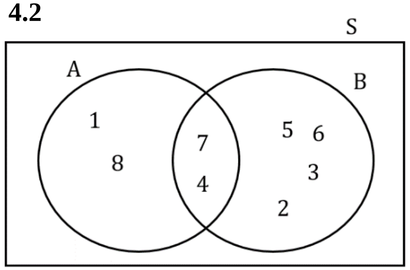
4.1



A and B are **inclusive** events → **P(A and B) ≠ 0**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{4}{10} + \frac{5}{10} - \mathbf{\frac{1}{10}}$$
$$= \frac{8}{10}$$

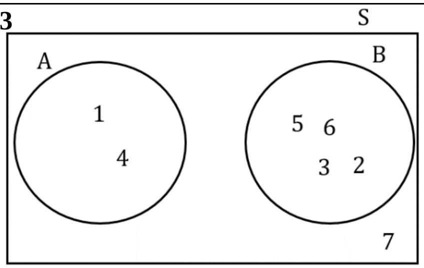
4.2



Event A and B is **exhaustive** → **P(A or B) = 1**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{4}{8} + \frac{6}{8} - \frac{2}{8}$$
$$= 1$$

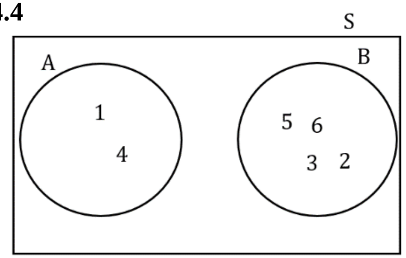
4.3



Events A and B are **mutually exclusive** → **P(A and B) = 0**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{2}{7} + \frac{4}{7} - \mathbf{0}$$
$$= \frac{6}{7}$$

4.4



Note:

$P(A') = P(\text{not}A)$

A and B are **complementary events** → (mutually exclusive and exhaustive)

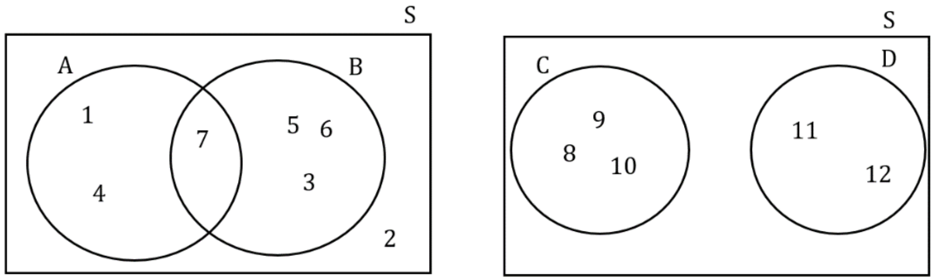
- $P(A) + P(B) = 1$
- $P(A) + P(\text{not } A) = 1$
- $P(A') = 1 - P(A)$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{2}{6} + \frac{4}{6} + \mathbf{0}$$
$$= \mathbf{1}$$

Mutually exclusive

exhaustive

CAN YOU? QUESTION 4



Use the above Venn-diagrams to determine the following:

4.1	P(D)	4.7	True or false: (Give a reason for your answer)
4.2	$P(A')$	4.7.1	Events A and B are inclusive events.
4.3	P (A and B)	4.7.2	Events A and B are exhaustive events
4.4	P (A or B)	4.7.3	Events C and D are mutually exclusive
4.5	P (C and D)		
4.6	P (C or D)	4.8	Which of the events above are complementary events?

SOLUTIONS

4.1	$P(D) = \frac{2}{5} = 0{,}4$		
4.2	$P(A') = \frac{4}{7}$	4.7.1	True $P(A \text{ and } B) \neq 0$
4.3	$P(A \text{ and } B) = \frac{1}{7}$	4.7.2	False $P(A \text{ or } B) \neq 1$
4.4	$P(A \text{ or } B) = \frac{6}{7}$	4.7.3	True $P(A \text{ and } B) = 0$
4.5	$P(C \text{ and } D) = 0$		
4.6	$P(C \text{ or } D) = 1$	4.8	C and D

NEW CONCEPTS AND SKILLS – GR 11

- CAPS GR 11: Dependent and independent events. (b) **Venn diagrams** or contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).

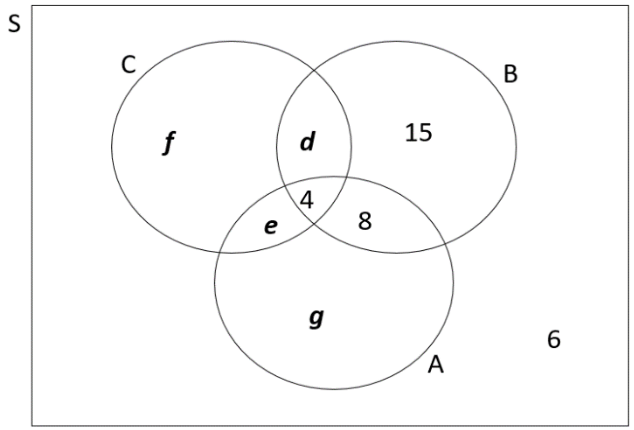
- NOTE: In grade 11 Venn-diagrams with 3 events will be assessed. (not only 2 as in GR 10)

EXAMPLE 5:

Research has been conducted on alcohol-based management. Information collected at the traffic authorities of 54 countries on the methods used to test the alcohol level of a person is summarized below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries do not use either of these methods.

On the right is a partially completed Venn diagram representing the information above.



5.1 Use the given information and the Venn diagram to determine the values of d, e, f and g.

5.2 For a randomly selected country, calculate:

5.2.1 P(A and B and C)

5.2.2 P(A or B or C)

5.2.3 P(only C)

5.2.4 P(uses exactly two methods)

SOLUTIONS

5.1 $d = 5$ $e = 4$ $f = 7$ $g = 5$

5.2.1 $\text{P(A and B and C)} = \frac{4}{54} = \frac{2}{27}$

5.2.2 $\text{P(A or B or C)} = \frac{48}{54} = \frac{8}{9}$

5.2.3 $\text{P(only C)} = \frac{7}{54}$

5.2.4 $\text{P(uses exactly two methods)} = \frac{5+4+8}{54} = \frac{17}{54}$

NEW CONCEPTS AND SKILLS – GR 11

CAN YOU? **QUESTION 5**

A survey is conducted with a group of 50 learners to find out what is more popular at the school tuck-shop. They are asked if they usually buy toasted sandwiches (T), wraps (W) or burgers (B). They can choose none, one, two or three of the meals.

- 21 chose sandwiches
- 20 chose wraps
- 13 chose burgers
- 3 chose all three options
- 8 chose wraps and burgers
- 4 chose only burgers
- 5 chose toast and wraps

5.1 Complete a Venn diagram to represent this information

5.2 How many people did not buy wraps, toasted sandwiches or burgers?

5.3 Calculate the probability that a learner selected at random from this survey:

5.3.1 buys wraps and toasted sandwiches, but not burgers.

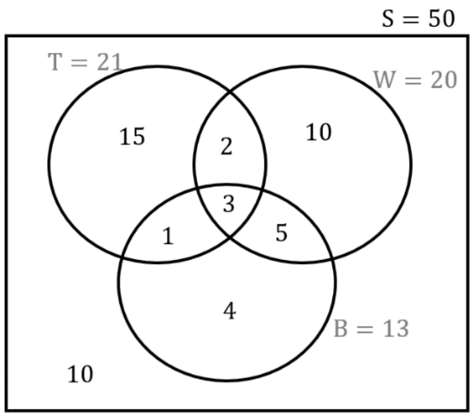
5.3.2 had only burgers.

5.3.3 had at least one meal.

5.3.4 did not have toast sandwiches.

SOLUTIONS

5.1



5.2 10

5.3.1 $\text{P(wraps and toast)} = \frac{2}{50} = \frac{1}{25}$

5.3.2 $\text{P(only burgers)} = \frac{4}{50} = \frac{2}{25}$

5.3.3 $\text{P(at least one meal)} = 1 - \frac{10}{50} = \frac{40}{50} = \frac{4}{5}$

5.3.4 $\text{P}(\text{T}') = \frac{10+5+4+10}{50} = \frac{29}{50}$

NEW CONCEPTS AND SKILLS – GR 11

CAN YOU? **QUESTION 6**

There are 240 boys in grade 11.
The following information on participation in school sport was collected:

- 122 boys play rugby (R)
- 58 boys play hockey (H)
- 96 boys play cricket (C)
- 16 boys take part in all three sports
- 22 boys take part in rugby and hockey
- 26 boys take part in cricket and hockey
- 26 boys do not participate in any of the sports.
- Let the number of learners playing rugby and cricket be x.

6.1 Complete a Venn diagram to represent this information

6.2 Calculate the value of x.

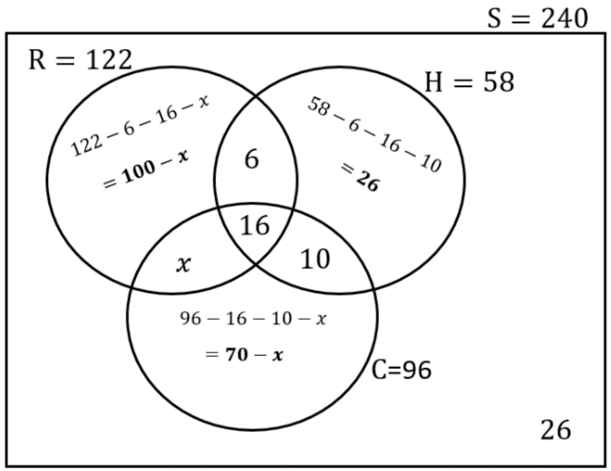
6.3 Calculate the probability that a learner who is chosen involuntarily:

6.3.1 Will play only hockey.

6.3.2 Does not play cricket.

SOLUTIONS

6.1



6.2 $x + 16 + 6 + 10 + 100 - x + 26 + 70 - x + 26 = 240$

$$254 - x = 240$$

$$x = 14$$

6.3.1 $P(\text{only hockey}) = \dfrac{26}{240} = \dfrac{13}{120}$

6.3.2 $P(\text{not C}) = \dfrac{86 + 26 + 6 + 26}{240} = \dfrac{144}{240} = \dfrac{3}{5}$ **OR**

$P(\text{not C}) = 1 - \dfrac{96}{240} = \dfrac{3}{5}$

ACTIVITIES / ASSESSMENT	*Mind the Gap* • *P145-156* • *p 157 Activity 3*	*Siyavula* • *P402-410* • *P419-p425*	*Mind Action Series* • *P265 Exercise 1* • *P274 Exercise 4*

CONSOLIDATION:

A Venn diagram is a visual tool used to show how events overlap.
Each region in a Venn diagram represents an event and could contain either the outcomes in the event, the number of outcomes in the event or the probability of the event

- The **addition rule** (also called the sum rule) for any 2 events, A and B is

$$\mathbf{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

This rule relates the probabilities of 2 events with the probabilities of their union and intersection.

- The addition rule for **2 mutually exclusive** events is

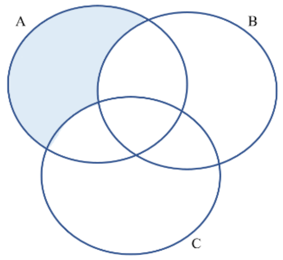
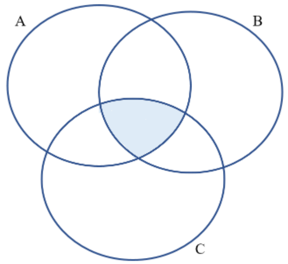
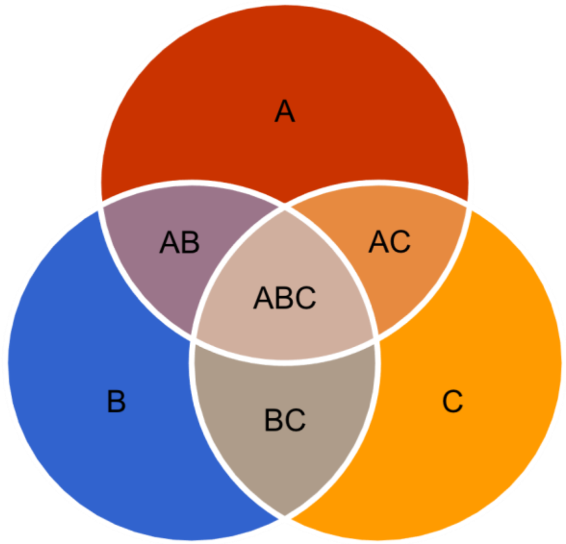
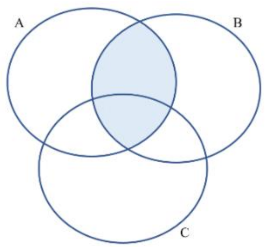
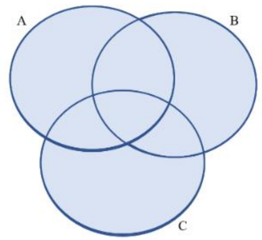
$$\mathbf{P(A\ or\ B) = P(A) + P(B)}$$

This rule is a special case of the previous rule. Because the events are mutually exclusive, **P(A and B) = 0.**

- The **complementary rule** is

$$\mathbf{P(not\ A) = 1 - P(A)}$$

Since A and (not A) are mutually exclusive and exhaustive events.

WORDS & SYMBOLS	**VENN-DIAGRAM**	**WORDS & SYMBOLS**	**VENN-DIAGRAM**	
Only A		Al A and B and C A ∩ B ∩ C		
A and B A∩ B		At least one A or B or C A ∪ B ∪ C		

VALUES:

Venn diagrams are commonly associated with education. They are frequently used in mathematics to understand set theory and also used to do various comparisons in the classroom. However, there are many other uses of Venn diagrams that you can take advantage of during your daily routines. The power of Venn diagram lies in its simplicity. They are great for comparing things in a visual manner and to quickly identify overlaps.