SUBJECT and GRADE	Mathematics Grade 12	
TERM 2	Week 6	
TOPIC	Application of Calculus	
AIMS OF LESSON	• Sketch the derivative of a cubic function • Discuss the connection between a function and it's derivative • The second derivative and application of Concavity	
RESOURCES	**Paper based resources**	**Digital resources**
	Please go to the Calculus section in your Mathematics textbook. Mind Gap: Page 139	https://www.youtube.com/watch?v=DdCfufivnjI https://www.youtube.com/watch?v=Zq7g1nc2MJ8 https://www.youtube.com/watch?v=mamH094uw_U

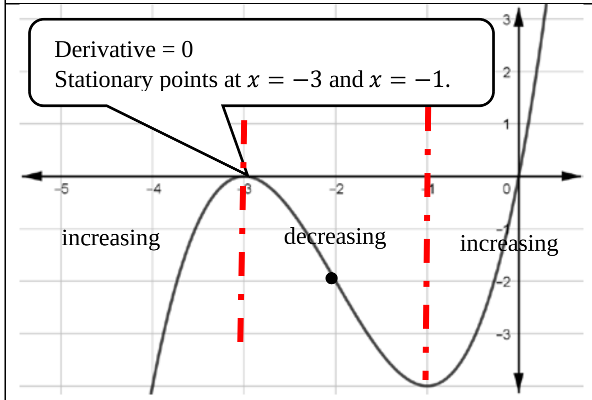
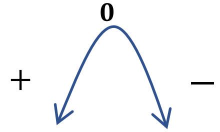
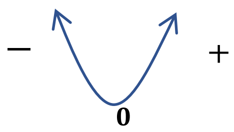
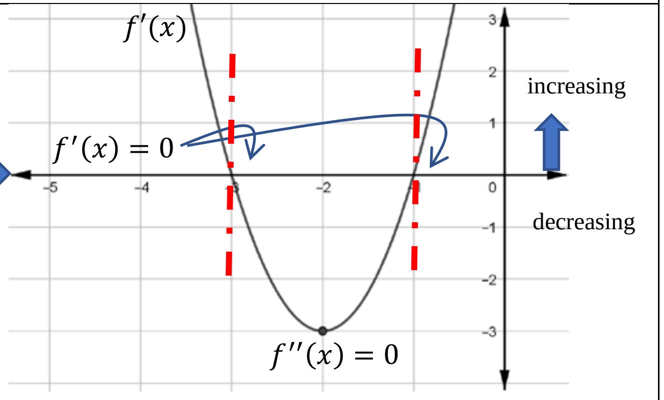
INTRODUCTION

Sketched below is the graph of $f(x) = x^3 + 6x^2 + 9x$

The derivative of $f(x)$, $f'(x) = 3x^2 + 12x + 9$ is drawn on the right below:

How do these two graphs relate?

Look especially at the stationary points!

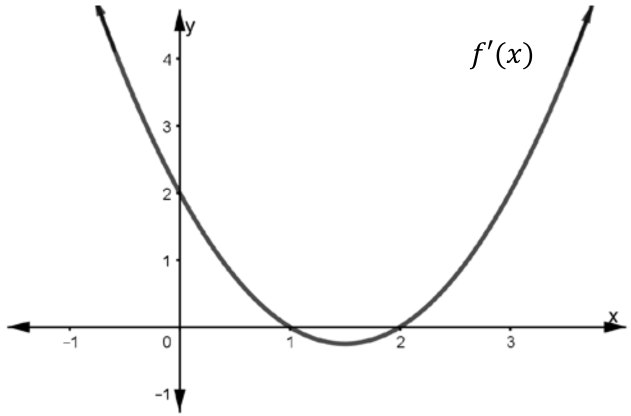
The function $f(x) = x^3 + 6x^2 + 9x$	**Notes**	The derivative $f'(x) = 3x^2 + 12x + 9$
	Local Maximum:  Local Minimum: 	
Local Max at $x = -3$ Local Min at $x = -1$	The derivative is positive above the x −axis and negative below the x −axis.	Point of inflection is on the A/S of the parabola. Midway between the 2 stationary points

CONCEPTS AND SKILLS

Now we can do the reverse!

Examples

1. In the sketch below, the graph $y = ax^2 + bx + c$ represents the derivative of a function f, where f is a cubic function.



(a) Write down the $x-$ coordinates of the stationary points.

(b) State whether these points is a local minimum or local maximum.

(c) Determine the x −coordinate of the point of inflection.

(d) Hence draw a rough sketch of $f(x)$ showing the main points

Solution:

(a) $f'(x) = 0$

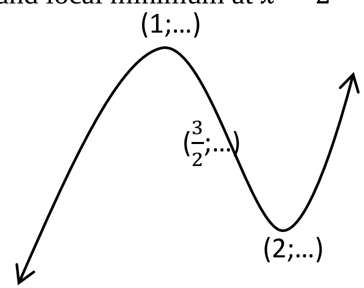
$x = 1$ and $x = 2$

(b) Local maximum at $x = 1$ and local minimum at $x = 2$

(c) Point of inflection:

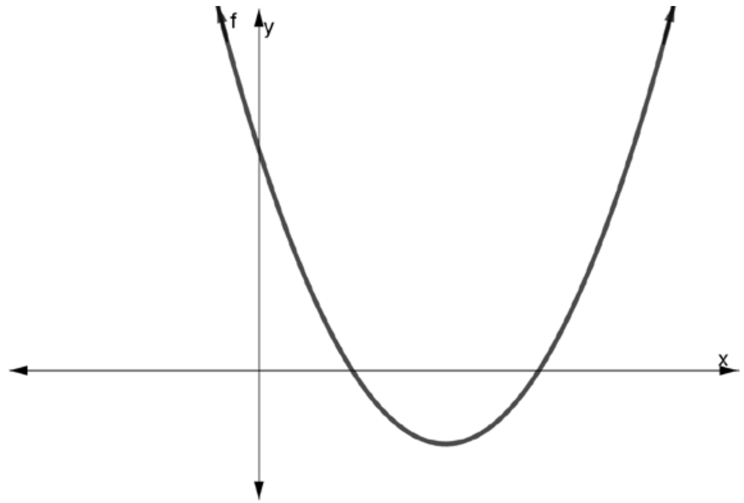
$x = \frac{3}{2}$

(d)



Can you?

1. The graph of, $y = ax^2 + bx + c$, below represents the derivative of f. It is given that $f'(1) = 0$, $f'(3) = 0$ and $f'(0) = 6$.



(a) Write down the x −coordinate of the stationary points of f.

(b) For which value(s) of x is f strictly decreasing?

(c) Explain at which value of x ,the stationary point of f will be a local minimum.

(d) Determine the x −coordinate of the point of inflection of f

(e) For which value(s) of x is f concave up?

Answers:

(a) $x = 1 \; en \; x = 3$

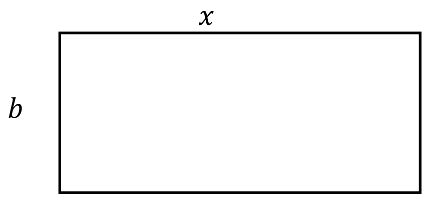
(b) $1 < x < 3$

(c) $x = 3$ local minimum

(d) $x = 2$

(e) $x > 2$

2. A rectangle has a perimeter of 160m. Find the length and width of the rectangle to give maximum possible area.



Solution:

Perimeter of rectangle:

$P = 2(l + b)$

Let $l = x$

$x + b = 80$

$b = 80 - x$

$Area = l \times b$

$= x(80 - x)$

$A = -x^2 + 80x$

$\frac{dA}{dx} = -2x + 80$

$-2x + 80 = 0$

$-2x = -80$

$x = 40$

$l = 40 \; and \; b = 80 - 40 = 40$

- Start with given information
- Write information i.t.o variable
- Use information to create the equation

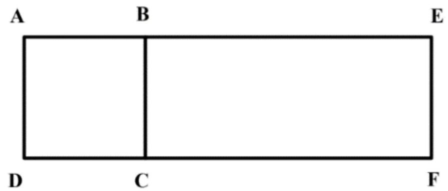
For max area: $\frac{dA}{dx} = 0$

Can you?

QUESTION 9 [Feb 2017]

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.

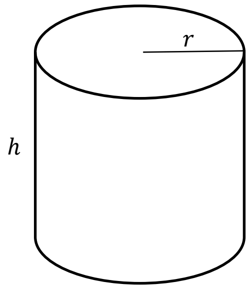
Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum.



Answer:

$x = 4$

3. A softdrink can has a volume of 340 cm^3, a height h cm and a radius of r cm.



(a) Express h in terms of r

(b) Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$

(c) Determine the radius of the can that will ensure the surface are is a minimum.

Solution:

(a) $Volume = area\ of\ base \times h$

$\pi r^2 h = 340$

$h = \dfrac{340}{\pi r^2}$

(b) $Surface\ area = 2\pi r \times h + 2\pi r^2$

$A = 2\pi r \times \dfrac{340}{\pi r^2} + 2\pi r^2$

$= \dfrac{680}{r} + 2\pi r^2$

(c) For min Area :

$\dfrac{dA}{dr} = 0$

$-680r^{-2} + 4\pi r = 0$

$4\pi r^3 = 680$

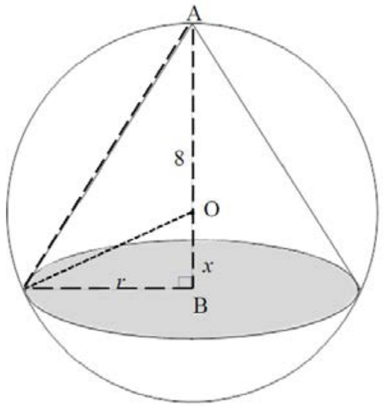
$r^3 = \dfrac{680}{4\pi} = 54.11$

$r = 3.78$

Can you?

QUESTION 9 [May/June 2019]

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB = x.



Volume of sphere $= \dfrac{4}{3}\pi r^3$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$ (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

Answers:

9.1 Volume$= \dfrac{2048\pi}{3} = 2144.66$

9.2 $r^2 + x^2 = 64$

9.3 Max volume van cone: Volume van sphere

$\dfrac{Vol\ cone}{Vol\ sphere} = \dfrac{\frac{\pi}{3}(\frac{512}{9})(\frac{32}{3})}{\frac{2048\pi}{3}} = \dfrac{8}{27}$

ACTIVITIES/ ASSESSMENT	Mind Action Series	Platinum	Classroom Mathematics	Via Africa
	Ex: 10-12; Pg: 202-208	Ex: 14; Pg: 173	Ex: 9.4; Pg: 230	Ex: 13; Pg: 194

CONSOLIDATION

- Remember to start with the given information
- The formula or expression must have only one variable. We need for differentiate w.r.t that variable.
- For max/min, you need to solve the equation: derivative = 0.