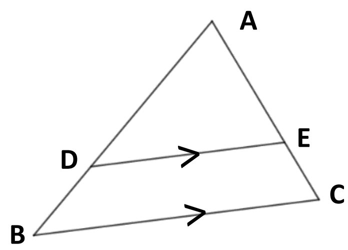
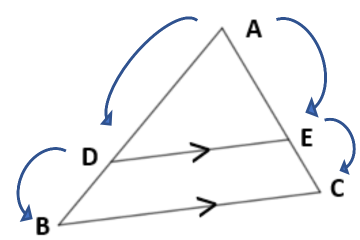
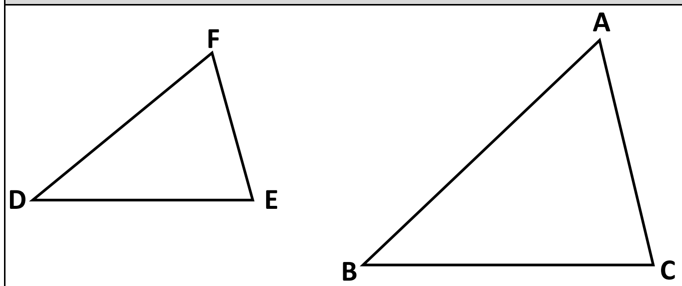
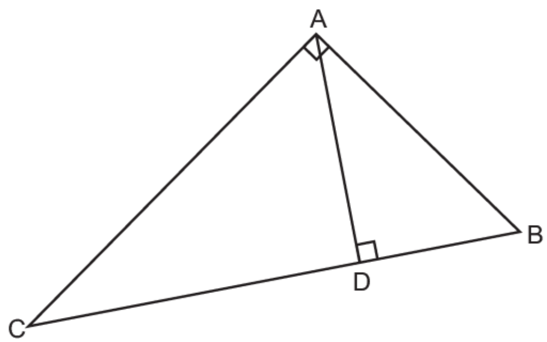
SUBJECT and GRADE	Mathematics Grade 12
TERM 1	Week 6
TOPIC	Euclidean Geometry Pythagorean Theorem and mixed problems
AIMS OF LESSON	• Use similarity to prove the Theorem of Pythagoras • Apply knowledge to problems • Answer riders using a combination of theorems
RESOURCES	***Paper based resources*** Go to this section in your textbook.

INTRODUCTION: Up to this stage you should know the following facts:

	If	then
Theorem 1: Proportionality	 DE ∥ BC	 $\frac{AD}{DB}=\frac{AE}{EC}$
Theorem 2: Similarity	 Δ FDE ||| Δ ABC	$\frac{DF}{AB}=\frac{FE}{AC}=\frac{DE}{BC}$

CONCEPTS AND SKILLS

We will be using the above skills to prove the following:
Let's look at the following special case.



How many triangles are there?
ΔABC; ΔABD and ΔADC

What is special in this case?

The **perpendicular** is drawn from the vertex of a right-angled triangle onto the **hypotenuse**

Let's see if these triangles are similar to each other.

In Δ ABC and Δ ABD:

1. $C\hat{A}B = A\hat{D}B$ [both $= 90°$]
2. $\hat{B} = \hat{B}$ [common]

$\therefore \ \Delta ABC \ ||| \ \Delta DBA$ $[\angle, \angle, \angle]$

$$\therefore \ \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$$

$\therefore \ AB^2 = BC \,.\, BD$

In Δ ABC and Δ DAC:

1. $C\hat{A}B = A\hat{D}C$ [both $= 90°$]
2. $\hat{C} = \hat{C}$ [common]

$\therefore \ \Delta ABC \ ||| \ \Delta DAC$ $[\angle, \angle, \angle]$

$$\therefore \ \frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{DC}$$

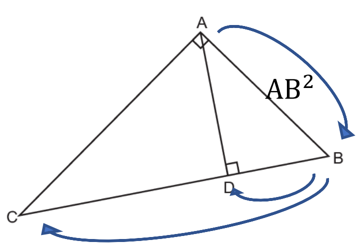
$\therefore \ AC^2 = BC \,.\, DC$

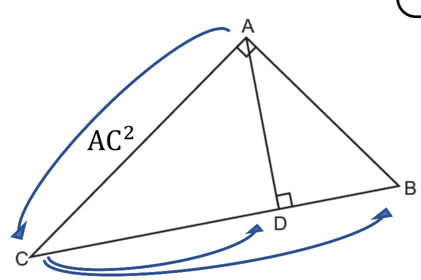
Δ ABD ||| ΔCAD [Both ||| Δ ABC]

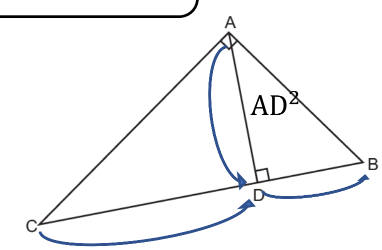
$$\therefore \ \frac{AB}{AC} = \frac{BD}{AD} = \frac{AD}{DC}$$

$\therefore \ AD^2 = BD.\,DC$

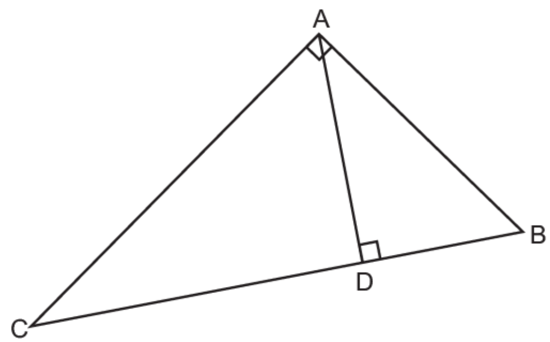
Look at the patterns formed in each case.







Using the results of the above, we can prove the Theorem of Pythagoras by using similarity.



The theorem of Pythagoras:

Required to prove:

$$BC^2 = AB^2 + AC^2$$

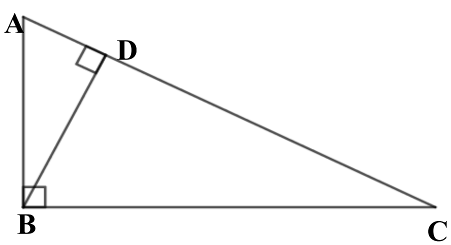
Given: Δ ABC with $\hat{A} = 90°$, and AD $\perp$ BC

Proof:

$AB^2 = BC.BD$
$AC^2 = BC.CD$

$AB^2 + AC^2 = BC.BD + BC.CD$
$= BC(BD + CD)$
$= BC(BC)$
$= BC^2$

Example 1:
In ΔABC, BD $\perp$ AC and AB $\perp$ BC.



Complete the following:

(a) Δ ABD ||| Δ …… ||| Δ …….
(b) Hence complete that:
$AB^2 =$
$BC^2 =$
$BD^2 =$
(c) If DC = 6 cm and AB = 4 cm, determine the length of AD
(d) Hence determine the length of BC.

Solution:

(a) Δ ABD ||| Δ ACB ||| Δ BCD

The angles must correspond. The right-angle is at the last letter in each case. **This simplifies matters!**

(b) $AB^2 = AD.AC$
$BC^2 = CD.CA$
$BD^2 = AD.DC$

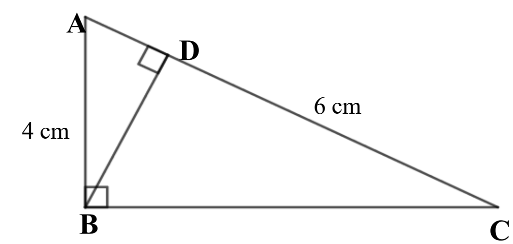
Δ **AB**D ||| Δ **A**C**B**

$$\frac{AB}{AC} = \frac{AD}{AB}$$

BC²?

Δ A**CB** ||| Δ **BC**D

(c)

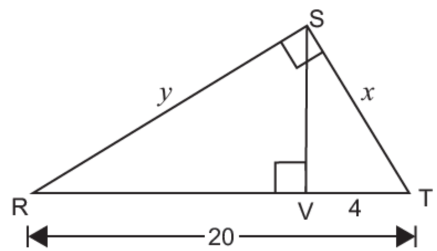
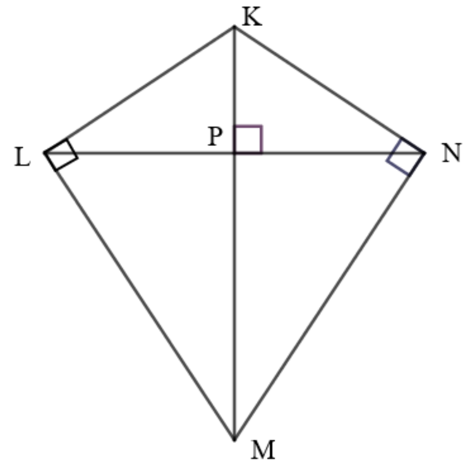


Let AD $= x$ units

$AB^2 = AD.AC$
$16\ cm^2 = x(x + 6)$
$36 = x^2 + 6x$
$x^2 + 6x - 36 = 0$
$(x + 8)(x - 2) = 0$
$x = -8 \ or \ x = 2$

AD $= 2\ cm$ **Length cannot be negative**

(d) $BC^2 = AC^2 - AB^2$ [Pyth]
$BC^2 = 8^2 - 4^2$
$BC = \sqrt{48} = 4\sqrt{3}\ cm$

CAN YOU?	
1. Find x and y.  **Solution:** $x = 4\sqrt{5}$ $y = 8\sqrt{5}$	2. In the accompanying diagram, KLMN is a kite with diagonals bisecting at P. $\hat{L} = \hat{N} = 90°$.  (a) Give a reason why Δ KLP ||| Δ KML. (b) Complete the following: $KL^2 = \ldots\ldots\ldots$ $LM^2 = \ldots\ldots\ldots$ $LP^2 = \ldots\ldots\ldots\ldots$ (c) Prove that: $\frac{PN^2}{KN^2} = \frac{MP}{MK}$ (d) Prove that: $KL^2 - KP^2 = KP \times PM$

Typical examination questions:
We are now going to apply **ALL** our knowledge on Proportionality and Similarity to the following problems.

Example 2:

In the diagram, DE is a tangent to the circle at E and DFG is a straight line.

DE = EF = FG and HF || DE.

It is further given that $\frac{\text{DF}}{\text{DE}} = y$. Let $\text{D}\hat{\text{E}}\text{F} = x$.

(a) Give, with reasons, THREE other angles equal to x.

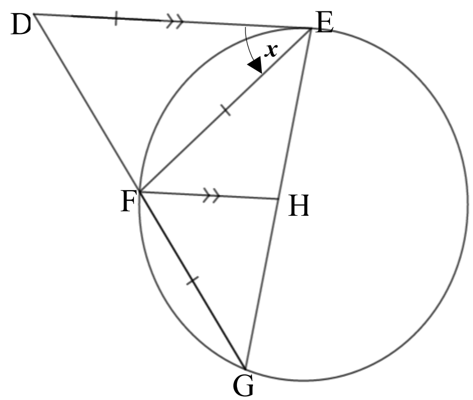
(b) Prove that:

(i) $\frac{\text{EH}}{\text{HG}} = y$

(ii) $\Delta\text{DGE} \,|||\, \Delta\text{DEF}$

(iii) $\text{DE}^2 = \text{DF. DG}$

(iv) $y^2 + y = 1$



Solution:

(a)

$\text{E}\hat{\text{F}}\text{H} = x$ [alt.∠s; DE || FH]

$\hat{\text{G}} = x$ [tan- chord theorem]

$\text{F}\hat{\text{E}}\text{G} = \hat{\text{G}} = x$ [∠s opposite equal sides]

(b)

(i) $\frac{\text{EH}}{\text{HG}} = \frac{\text{DF}}{\text{FG}}$ [line // 1 side of Δ]

$= \frac{\text{DF}}{\text{DE}}$ [FG = DE; given]

$= y$

(ii) In ΔDGE and ΔDEF:

1. $\hat{\text{D}}$ is common
2. $\hat{\text{G}} = \text{D}\hat{\text{E}}\text{F} = x$ [from (a) above]

∴ ΔDGE ||| ΔDEF [∠, ∠, ∠]

(iii)

∴ $\frac{\textbf{DG}}{\textbf{DE}} = \frac{\text{GE}}{\text{EF}} = \frac{\textbf{DE}}{\textbf{DF}}$ [ΔDGE ||| ΔDEF]

∴ $\text{DE}^2 = \text{DF. DG}$

(iv)

$\frac{\text{DE}}{\text{DF}} = \frac{\text{DG}}{\text{DE}}$ [from (ii)]

$\frac{1}{\text{y}} = \frac{\text{DF+FG}}{\text{DE}}$ [$\frac{\text{DF}}{\text{DE}} = y$ ∴ $\frac{DE}{DF} = \frac{1}{y}$]

$= \frac{\text{DF}}{\text{DE}} + \frac{\text{FG}}{\text{DE}}$ [FG = DE]

∴ $\frac{1}{y} = \text{y} + 1$

$\therefore y^2 + y = 1$

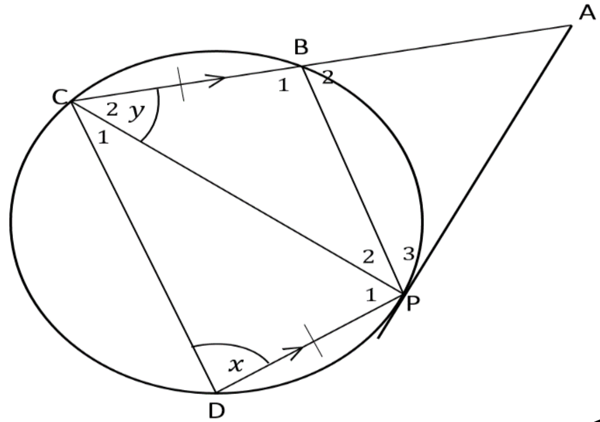
CAN YOU?

3.

AP is a tangent to the circle.

CB ∥ DP. CBA is a straight line.

Let $\hat{D} = x$ and $\hat{C}_2 = y$.



Prove, with reasons that:

(a) $\Delta APC \parallel\!| \Delta ABP$

(b) $AP^2 = AB \cdot AC$

(c) $\Delta APC \parallel\!| \Delta CDP$

(d) $AP^2 + PC^2 = AC^2$

Hint:

Use information from (a) and (c) to answer (d)

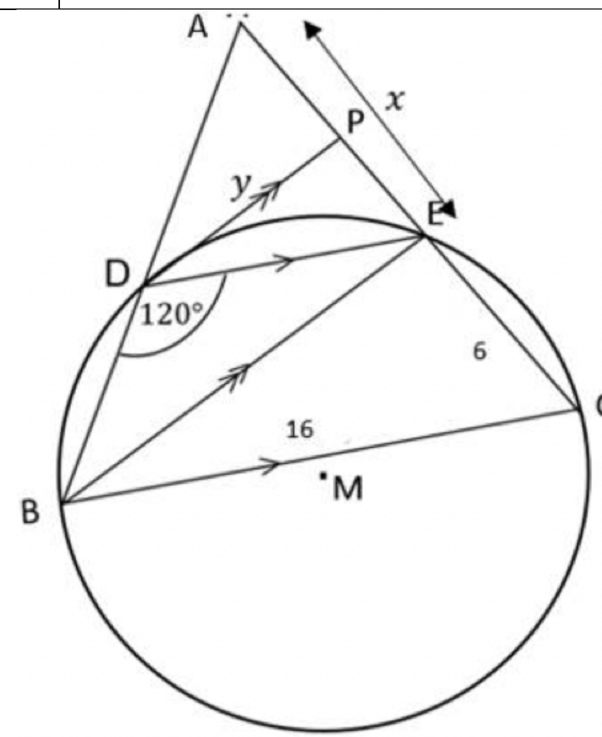
4.In the adjacent diagram is Δ ABC with points D, P and E on AB and AC such that,

DE ∥ BC and DP ∥ BE.

DECB is a cyclic quadrilateral and M is the centre of the circle.

Given: $B\hat{D}E = 120°$, EC= 6 $units$, BC = 16 $units$ and $\frac{AD}{DB} = \frac{5}{3}$

Let DP = y and AE = x



Determine, with reasons:

(a) The length of AE (x) **[$x = 10$]**

(b) $\frac{\text{area of } \Delta\, AEB}{\text{area of } \Delta\, ECB}$ **$\left[\frac{\text{area of } \Delta\, AEB}{\text{area of } \Delta\, ECB} = \frac{5}{3}\right]$**

(c). Prove that BE= 14

(d). Determine the length of DP (y) **[$y = 8,75$]**

ACTIVITIES/ASSESSMENT

Mind Action Series	Platinum	Clever	Classroom Mathematics	Siyavula
Exercise: 8 Page: 277	Exercise: 5 Page: 224	Exercise:11.5 Page: 303	Exercise: 11.4; 11.8 Page: 298	Exercise: 8.9 Page: 351

CONSOLIDATION

- Know your theorems
- Use different colours to highlight the given information
- The proof of Pythagorean Theorem cannot be tested in the examination
- The circle Geometry of Grade 11 can be integrated with these theorems.