SUBJECT and GRADE	Mathematics Grade 12	
TERM 2	Week 1	
TOPIC	Trigonometrical Equations – General Solution	
AIMS OF LESSONZ	Apply the Compound Angles and Double Angle Identities to solve equations.	
RESOURCES	***Paper based resources***	***Digital resources***
	Go to the Trigonometry topic in your textbook and go to the section involving Trigonometry. *(e.g. Mind the Gap: Unit 10.10 Compound and double angle identities Page 215 & Siyavula: Page 154; Chapter 4.4)*	https://bit.ly/2UREr8 https://www.siyavula.com *https://wcedeportal.co.za/eresource/80176* *https://wcedeportal.co.za/eresource/80191*
INTRODUCTION		

Let's solve an equation from Grade 11:

Solve for x, given $\sin x = 3\cos x$ where $\boldsymbol{x} \in (\mathbf{-360°; 360°})$

Solution:

$$\sin x = 3\cos x$$

$$\frac{\sin x}{\cos x} = \frac{3\cos x}{\cos x}$$ **[Divide by $\cos x$ to isolate the ratio]**

$$\tan x = 3$$ **[Tan is positive - CAST rule]**

Refence Angle: $x = 71{,}57°$

Quadrant 1: **Quadrant 3:**

$x = 71{,}57° + k.180°$ or $x = 180° + 71.57° + k.180 \ for\ k \in Z$

$\therefore \quad x = 251.57° + k.180° \ for\ k \in Z$

$\therefore \quad x = 71{,}57° + k.180°$

Specific Solution: Choose different values for k to determine solutions within the interval.

For $k = -2; -1; 0; 1; 2$

$Then$: $-288.43°$; $-108.43°$; $71.47°; 251.57°$

The basic rules to solve equations still apply:

- Isolate the ratio
- Use identities to simplify the equation
- Use the CAST rule to find the reference angle
- Write down the General Solution
- Select appropriate values for k to determine specific solutions

Note that with the tan equations both these equation give the same solutions. So for tan the general solution is one of them.

Only needed if restrictions are given!

Important Information

Compound Angles

$\sin(\alpha+\beta)=\sin\alpha\, cos\beta+\cos\alpha\sin\beta$; $\sin(\alpha-\beta)=\sin\alpha\, cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\, cos\beta-\sin\alpha\sin\beta$; $\cos(\alpha-\beta)=\cos\alpha\, cos\beta+\sin\alpha\sin\beta$

Double Angles

$$\cos 2\alpha = \begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

CONCEPTS AND SKILLS

Let's apply our knowledge of double and compound angles to these equations.

Example 1: Determine the general solution for:

$\sin 2x.\cos x+\cos 2x.\sin x=\cos 60°$ **[Compound rule on LHS]**

Solution:

$\sin(2x+x)=\frac{1}{2}$

$Ref\ Angle: 30°\quad [Quad\ 1]\quad or\ 180°-30\quad [Quad\ 2]$ **[CAST rule]**

Watch out for Special Angles

$2x+x=30°+k.360°\quad or\quad 2x+x=150°+k.360°$

$3x=30°+k.360°\quad or\quad 3x=150°+k.360°$

$\boldsymbol{x=10°+k.120°}\quad \boldsymbol{or}\quad \boldsymbol{x=50°+k.120°};\ \boldsymbol{k\in Z}$

Can you try these?

1. $\cos x=2\sin 75°.\cos 75°$
2. $\sin 2x=1-\cos 2x$
3. $\cos 2x=\sin x+1$

Solutions:

1. $x=60°+k.360; k\in Z\ or$
   $x=300°+k.360°; k\in Z$
2. $x=0°+k.360°; k\in Z$
   $x=45°+k.360°; k\in Z;$
3. $x=0°+k.360°; k\in Z$
   $x=270°+k.360°; k\in Z;$

Example 2: Determine the general solution of $\mathbf{2\sin 2x+3\sin x=0}$

Solution:

$2\sin 2x+3\sin x=0$ **[Double Angle - expand]**

$2(2\sin x.\cos x)+3\sin x=0$ **[Remove brackets]**

$4\sin x.\cos x+3\sin x=0$ **[factorise – common factor]**

$\sin x\,(4\cos x+3)=0$

$\sin x=0\quad or\quad \cos x=-\frac{3}{4}$ **[Be careful of the – sign]**

$Ref\ Angle:\ x=0°$ or $x=41.41°$

Quad 1: **[sin positive]**

$\boldsymbol{x=0°+k.360°;k\in Z}$

Quad 2: **[sin positive]**

$\boldsymbol{x=180°+k.360°;k\in Z}$

Quad 2: **[cos negative]**

$x=180°-41{,}41°=138{,}41°$

$\boldsymbol{x=138{,}41°+k.360°;\ k\in Z}$

Quad 3: **[cos negative]**

$x=180°+41{,}41°+k.360°; k\in Z$

$\boldsymbol{x=221{,}41°+k.360°;\ k\in Z}$

CAN YOU?

Determine the general solution of:

1. $\cos 2x+1=\cos x$
2. $\sin x-\sin 2x=0$
3. $\cos^2 x=2\sin x$

Solutions:

1. $x=90°+k.360°\ or 270°+k.360°; k\in Z$
   $x=60°+k.360°\ or\ 300°+k.360°; k\in Z$
2. $x=0°+k.360°\ or\ 180°+360°; k\in Z$
   $x=60°+k.360°\ or\ 300°+k.360°; k\in Z$
3. $x=90°+k.360°\ or 270°+k.360°; k\in Z$
   $x=9{,}46°+k.180°; k\in Z$

Example 3: Determine the general solution of $\cos 2x + \cos x = 0$

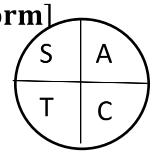
Solution:

$\cos 2x + \cos x = 0$ **[Double Angle- expand]**

$2\cos^2 x - 1 + \cos x = 0$ $\mathbf{[\cos^2 x - trinomial - standard\ form]}$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$ **[Factorise]**



$\cos x = \frac{1}{2}$ or $\cos x = -1$

$Ref\ Angle: x = 60°$ or $x = 180°$

Quad 1 **[cos positive]**

$x = 60° + k.360°\ ; k \in Z$

Quad 4 **[cos positive]**

$x = 360° - 60° + k.360°\ ; k \in Z$

$\boldsymbol{x = 300 + k.360°\ ; k \in Z}$

Quad 2 &3: [cos negative]

$\boldsymbol{x = 180° + k.360°\ ; k \in Z}$

CAN YOU?

Determine the general solution of:

1. $\cos 2x - 7\cos x - 3 = 0$
2. $\cos 2x + 4\sin^2 x = 5\sin x + 3$
3. $2\sin^2 x + \sin x = 3$

Solutions:

1. $x = 120° + k.360°$ *or* $x = 240° + k.360°; k\epsilon Z$
2. $x = 199° + k.360°$ *or* $x = 340{,}53° + k.360°; k \in Z$
3. $x = 90° + k.360°\ ; k\epsilon Z$

ACTIVITIES/ASSESSMENT

Mind Action Series	Platinum	Classroom Mathematics	Everything Mathematics	Clever	Via Africa
Ex: 7 &8; Pg: 129 & 131	Ex: 5-7 Pg: 97-100	Ex: 5.9 Pg: 147	Ex: 4.4 Pg: 160	Ex: 6.4-6.5 Pg: 129-130	Ex 5 Pg: 127

CONSOLIDATION

- *It is important to remember that the sine and cosine function repeats every* 360°*but the tan function repeats every* 180°
- *The CAST diagram plays an important role.*
- *Remember the reference angle is always in the first Quadrant. Use this information to find the angle in the other quadrants.*
- *Ignore the negative sign when determining the reference angle. The sign will be the indication of the quadrant. In which the solution is. CAST diagram.*
- *Practice!*