SUBJECT and GRADE	Mathematics Grade 12	
TERM 2	Week 5	
TOPIC	The Cubic Function	
AIMS OF LESSON	• Sketch the Cubic function • Find the equation of a cubic function	
RESOURCES	**Paper based resources**	**Digital resources**
	Please go to the Calculus chapter in your Mathematics Textbook. Then go to the section on Cubic Graphs. Mind The Gap: Page 132	https://www.youtube.com/watch?v=XdVDYCwD6lM https://www.youtube.com/watch?v=3TJLOCYrTes https://youtu.be/8TaDfMpCshE

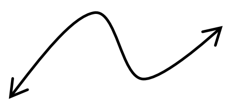
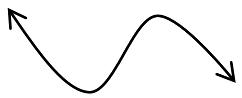
INTRODUCTION: Dear Grade 12 learner make sure you have worked through the week 2 lesson where you have learnt how to factorise a cubic function as well as the week 4 lesson where you were introduced to the rule of how to find the derivative of a function. All of that you will require in this lesson.

CONCEPTS AND SKILLS

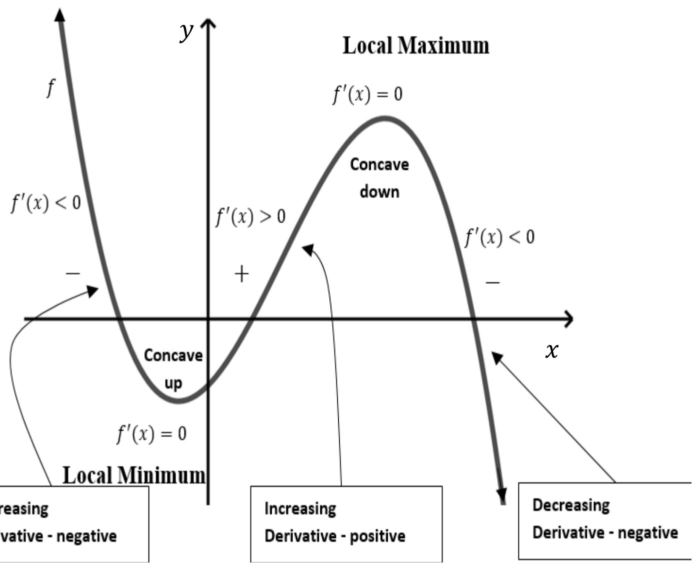
The standard form of the cubic function is $f(x) = ax^3 + bx^2 + cx + d$

1. Shape:

The value of "a" in the above equation influences the shape of the graph.

If $a > 0$ [positive value]	If $a < 0$[negative value]
	

Shape



2. Intercepts: Points where the graph intersects the axes

x –intercept	y –intercept
Let, $y = 0$, and solve for x by using the factor theorem.	Let, $x = 0$, and solve for y.

3. Turning Points, now referred to as STATIONARY POINTS

x –coordinate of stationary point	y –coordinate of stationary point
$f'(x) = 0$, and solve for x	Substitute the x-coordinate of stationary point into the original function to determine y.

To sketch a cubic function you will need to determine 1 – 3 above.

Increasing/Decreasing:

A function is **increasing** if as the x-values increase, the y-values are **increasing**.	A function is **decreasing** if as the x-values increases, the y-values are **decreasing**.
If $f'(x) > 0$, then the function is **increasing**	If $f'(x) < 0$, then the function is **decreasing**

Recall that, $f'(x)$, is the derivative of $f(x)$.
Also referred to as the first derivative.

CONCAVITY:

If $f''(x) > 0$, then the function is **concave up.**	If $f''(x) < 0$, then the function is **concave down.**

$f''(x)$: this is referred to as the second derivative.
$f''(x)$: is obtained by taking the derivative of, $f'(x)$.

POINT OF INFLECTION.

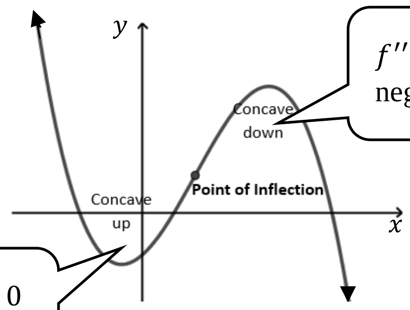
Point of inflection is the point where the graph changes from concave up to concave down and vice versa.

To **determine** the point of inflection of a cubic function, you need to determine the x- coordinate and the y-coordinate of the point.

x **−coordinate** of Point of Inflection	y **−coordinate** of Point of Inflection
$f''(x) = 0$ and solve for x	Substitute the x-coordinate of the point of inflection into the original function to determine y.

To **Show that** , "$x = a$", where, $f''(a) = 0$, is a point of inflection, proceed as follows:

Take, a number to the right of "a", say "$a + 1$", and determine, if $f''(a+1) > 0$ or $f''(a+1) < 0$
Take, a number to the left of "a", say "$a - 1$", and determine, if $f''(a-1) > 0$ or $f''(a-1) < 0$
If the sign of, $f''(a+1)$ and $f''(a-1)$ is different then you are able to conclude that the concavity of the function changes at, "$x = a$", therefore you can conclude that, "$x = a$", is a point of inflection.

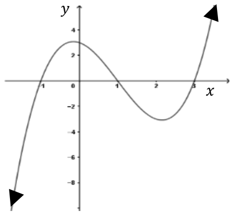
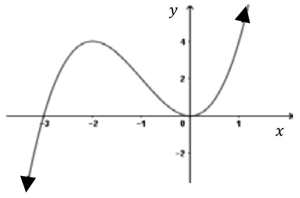
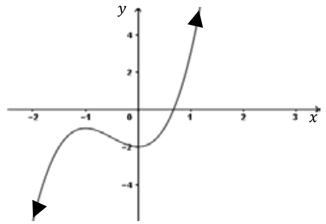
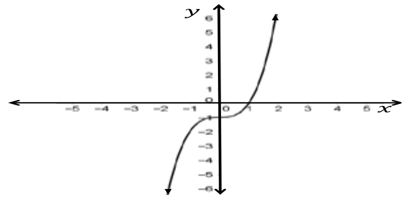


$f''(x) < 0$
negative

$f''(x) > 0$
positive

SKETCHING CUBIC GRAPHS

Below are 4 types of cubic sketch options. Note for each type below, “ $a > 0$”. The same four types can be drawn where, “ $a < 0$”.

Type 1: $y = x^3 - 3x^2 - x + 3$	**Type 2:** $y = x^3 + 3x^2$	**Type 3:** $y = 2x^3 + 3x^2 - 2$	**Type 4:** $y = x^3 - 1$
			
Three x –intercepts/roots **Two** turning points	**Two** x –intercepts/roots **Two** turning points **Note:** We have 2 equal roots at $x = 0$	**One** x –intercept/root **Two** points turning	**One** x –intercept **One** stationary point (not turning point but **Point of inflection**)

Example 1: Given: $f(x) = 2x^3 - 5x^2 - 4x + 3$

a) **Sketch the graph of $f(x)$** **b) Determine the point of inflection of f.**

Solution:

(a) Shape: $a = 2$ thus 

Intercepts:

x – intercepts:

$2x^3 - 5x^2 - 4x + 3 = 0$

$(x + 1)(2x^2 - 7x + 3) = 0$

$(x + 1)(2x - 1)(x - 3) = 0$

$x = -1 \ or \ x = \frac{1}{2} \ or \ x = 3$

Factor theorem:
Let $x = -1$:
$f(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$
$= -2 - 5 + 4 + 3 = 0$
Therefore $x + 1$ is a factor

Use long **division** or **inspection** to determine the quadratic factor.

Roots

Let $x = 0$:

y – intercepts: $y = 3$

Stationary Points: $f'(x) = 0$

$6x^2 - 10x - 4 = 0$

$2(3x^2 - 5x - 2) = 0$

$2(3x + 1)(x - 2) = 0$

$x = -\frac{1}{3} \quad or \ x = 2$

y –coordinates of stationary points

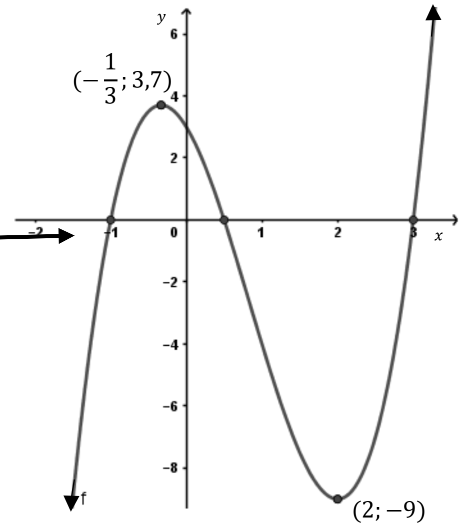
$f\left(-\frac{1}{3}\right) = 2\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 4\left(-\frac{1}{3}\right) + 3$

$= \frac{100}{27} = 3{,}7$

$f(2) = 2(2)^3 - 5(2)^2 - 4(2) + 3 = -9$

Stationary points: $\left(-\frac{1}{3}; 3{,}7\right)$ *and* $(2; -9)$

Sketch



(b) Point of inflection:

$f'(x) = 6x^2 - 10x - 4$

$f''(x) = 12x - 10$

$f''(x) = 0$

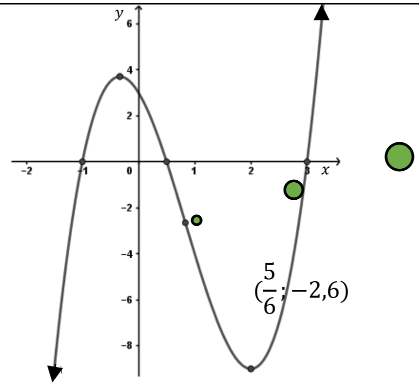
$12x - 10 = 0$

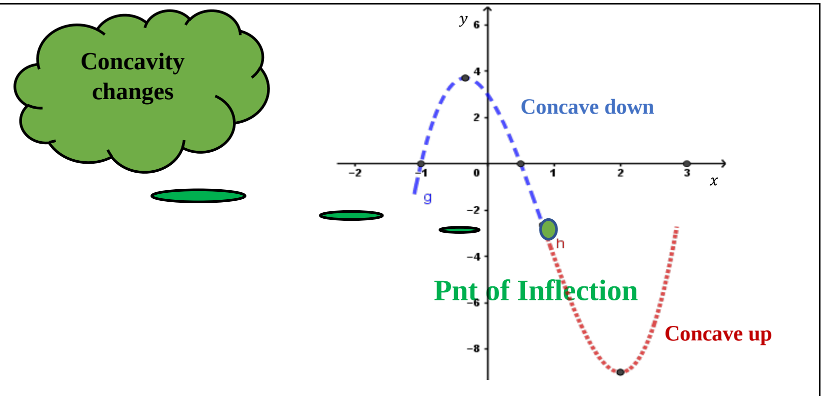
$12x = 10$

$x = \frac{10}{12} = \frac{5}{6}$

$f\left(\frac{5}{6}\right) = 2\left(\frac{5}{6}\right)^3 - 5\left(\frac{5}{6}\right)^2 - 4\left(\frac{5}{6}\right) + 3 = -2{,}6$

$\therefore$ $\left(\frac{5}{6}; -2{,}6\right)$ is the point of inflection



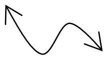


Example 2:

Sketch the function $g(x) = -x^3 + 6x^2 - 9x$

Solution:

Shape: $a = -1$ thus



Intercepts:

***x* − intercepts**: let $y = 0$

$-x^3 + 6x^2 - 9x = 0$

$-x(x^2 - 6x + 9) = 0$

$-x(x-3)(x-3) = 0$

$x = 0 \quad or\ x = 3$

***y* −intercept**: let $x = 0$ then

$y = 0$

Stationary Points:

$g'(x) = -3x^2 + 12x - 9$

$g'(x) = 0$

$-3x^2 + 12x - 9 = 0$

$-3(x^2 - 4x + 3) = 0$

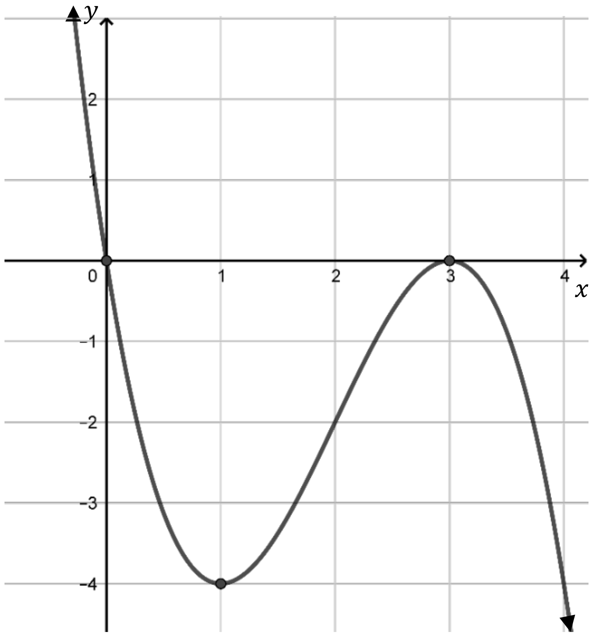
$-3(x-3)(x-1) = 0$

$x = 3 \qquad or\ x = 1$

***y* −coordinate of stationary point**:

$g(3) = -(3)^3 + 6(3)^2 - 9(3) = 0$

$g(1) = -(1)^3 + 6(1)^2 - 9(1) = -4$



$\therefore$ Coordinates of stationary points: $(3; 0)$ *and* $(1; -4)$

CAN YOU?

Sketch the following cubic functions:
[Show all intercepts and stationary point]

(a) $y = x^3 + 14x^2 - 49x + 36$

(b) $y = x^3 - 5x - 8x + 12$

(c) $y = x^3 - 4x^2 + 4x$

(d) $y = x(x-1)(x+1)$

Finding the equation of a cubic function

Given three roots and one other point

Use the formula:

$y = a(x - r_1)(x - r_2)(x - r_3)$ where $r_1\ ; r_2$ and r_3 are the roots.

- Substitute the roots and simplify as far as possible.
- Substitute the other point to determine the value of a.

Example 3

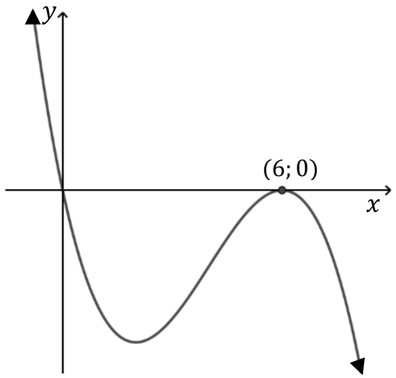
Given:

$\boldsymbol{f(x) = -x^3 + ax^2 + bx + c}$

and the sketch,

(a) find the values of a, b and c.

(b) Determine the coordinates of A, the local minimum of the curve.



Solution:

(a) $y = a(x - r_1)(x - r_2)(x - r_3)$

$f(x) = -(x - 0)(x - 6)(x - 6)$

$f(x) = -x(x^2 - 12x + 36)$

$f(x) = -x^3 + 12x^2 - 36x$

$\therefore\ a = 12;\quad b = -36\quad and\ c = 0$

> $a = -1$
> Roots: 0; 6; 6
> Equal roots at 6

> $y = 0$

(b) $f(x) = -x^3 + 12x^2 - 36x$

$f'(x) = -3x^2 + 24 - 36$

$f'(x) = 0$

$-3x^2 + 24x - 36 = 0$

$-3(x^2 - 8x + 12) = 0$

$-3(x - 6)(x - 2) = 0$

$x = 6$ or $x = 2$

y - value at $x = 2$.

$f(x) = -x^3 + 12x^2 - 36x$

$f(x) = -(2)^3 + 12(2)^2 - 36(2) = -32$

$\therefore$ **A$(2; -32)$**

> The turning point at, $x = 6$ was given.
> Therefore x –coordinate of A must be2.

Given the turning point and two other points

Start with:

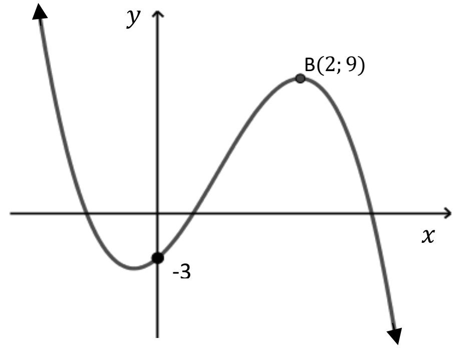
- Derivative = 0 at the turning point
- Substitute other points

Example 4

The sketch below is defined by

$\boldsymbol{p(x) = ax^3 + 5x^2 + 4x + d}$.

Determine the values of a and d.



Solution:

$y = ax^3 + 5x^2 + 4x + d$

$\frac{dy}{dx} = 3ax^2 + 10x + 4$

$\frac{dy}{dx} = 0$

$3ax^2 + 10x + 4 = 0$

$3a(2)^2 + 10(2) + 4 = 0$

$12a + 20 + 4 = 0$

$12a = -24$

$a = -2$

$\therefore\ y = -2x^3 + 5x^2 + 4x + d$

$d = -3$

$y = -2x^3 + 5x^2 + 4x - 3$

> The gradient at the turning point is zero.

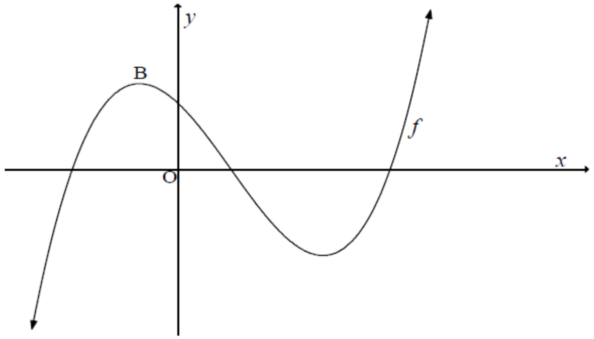
> x value at turning point is 2. Substitute into derivative

> Graph cuts y –axis at -3

CAN YOU? Question from Feb/March 2018 – Paper1

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1	Calculate the values of b, c and d.	(4)
9.2	Calculate the x-coordinate of B, the maximum turning point of f.	(4)
9.3	Determine an equation for the tangent to the graph of f at $x=-1$.	(4)
9.4	In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch.	(3)
9.5	For which value(s) of x is $f(x)$ concave upwards?	(2)
		[17]

Memo

9.1 $f(x)=(x+2)(x-1)(x-4)$
$f(x)=x^3-3x^2-6x+8$
$b=-3$; $c=-6$; $d=8$

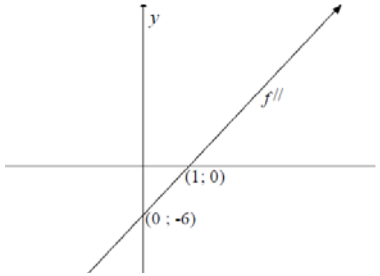
9.2 $f'(x)=0$
$3x^2-6x-6=0$
$x=-0{,}73$

Use formula

9.3 $f'(-1)=3(-1)^2-6(-1)-6=3$
$f(1)=(-1)^3-3(-1)^2-6(-1)+8=10$
$y-10=3(x+1)$
$y=3x+13$

$y-y_1=m(x-x_1)$

9.4 $f''(x)=6x-6$



9.5 $f''(x)>0$
$6x-6>0$
$6x>6$
$x>1$

ACTIVITIES/ ASSESSMENT	Topic	Mind Action Series	Platinum	Classroom Mathematics	Via Africa
	Sketching	Ex: 6; Pg: 188	Ex: 11&12; Pg: 162	Ex: 9.1& 9.2; Pg: 216;220	Ex: 11; Pg: 185
	Finding equations	Ex: 7;Pg: 190	Ex: 13 ; Pg: 166	Ex: 9.3; Pg: 1224	Ex: 11; Pg: 224

CONSOLIDATION

- Remember to follow the steps when sketching: Shape; Intercepts; Stationary points
- The **derivative** at the stationary point is 0.
- If the concavity changes at a point where "$x=a$" and, $f''(a)=0$, then the point where "$x=a$" is a point of inflection.