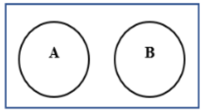
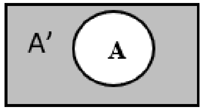
SUBJECT and GRADE	Mathematics Grade 12	
TERM 3	*Week 4*	
TOPIC	Probability: The Counting Principal	
AIMS OF LESSON	• Explain the fundamental counting principle • Apply the fundamental counting principle in different problem settings • Use of factorial notation.	
RESOURCES	***Paper based resources***	***Digital resources***
	Mind the Gap: Unit 8; Page 14 Your textbook	https://www.youtube.com/watch?v=GB6emV1cHW4 https://www.youtube.com/watch?v=ut61zNQ92dw
INTRODUCTION		

Summary of Grade 11 concepts that are very important:

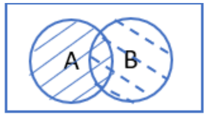
- The addition rule for **mutually exclusive** events:
  Two events are mutually exclusive
  if the two events cannot occur at the same time.
  $P(A \text{ and } B) = 0$ **[No intersection]**



- **The complementary rule:**
  $P(not\ A) = P(A') = 1 - P(A)$



- The probability identity:
  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

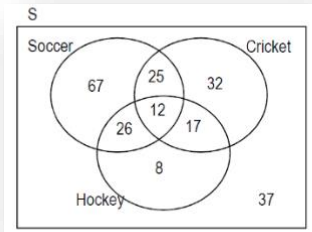


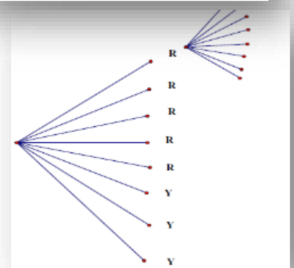
- **Dependent and independent events**
  Independent events - the outcome of the second event is not affected by the outcomes of the first event.

  The product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$

Notation
$P(A)$: Probability of event A occurring
$P'(A)$: probability of event A NOT occurring
∪: union/ **OR**
∩: intersection/ **AND**
$$P(E) = \frac{n(E)}{n(S)}$$

- Use **Venn diagrams** to solve Probability problems
- Use of **tree diagrams** to determine the probability of consecutive or simultaneous events which are not necessarily independent



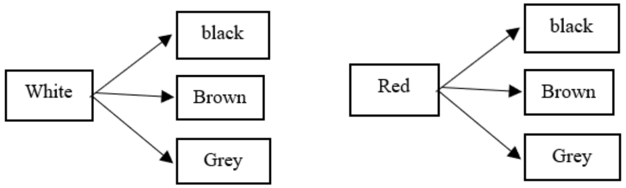


CONCEPTS AND SKILLS

Let's look at our first problem:

How many different outfits can be combined using a shirt and a pair of pants from 2 shirts and 3 pairs of pants.
Shirts: red and blue
Pants: black, brown and grey.

If we use a diagram



Solution:
White/black; White/brown; White/grey
Red/black; Red/brown; Red/Grey
Therefore **6** possible outcomes.

We can notice the 2 shirts that could be matched with 3 pairs of pants.

Therefore: **$2 \times 3 = 6$ possible outcomes**

You can just do the simple calculation:
Total Choices = 2 × 3 = 6

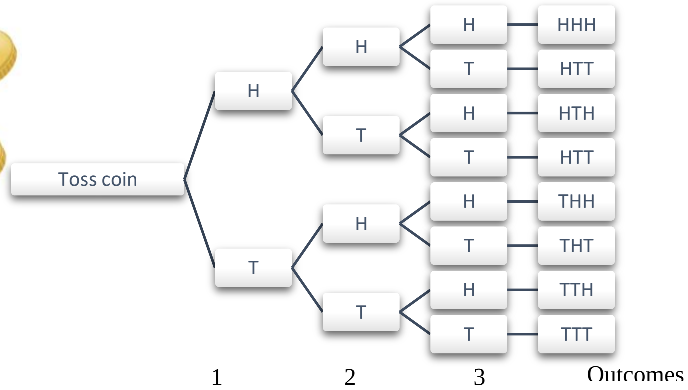
The Counting Principle
help you to count the possible outcomes without drawing a tree diagram.

The fundamental counting principle states:
Suppose there are n ways to make a choice, and for each of these there are m ways to make a second choice then the number of possible outcomes will be equal to $m \times n$ ways.

Let's test this rule: can use the fundamental counting principle to find the number of outcomes when a coin is tossed three times.

- There are 2 ways to get the first outcome (H or T).
- There are 2 ways to get the second outcome (H or T).
- There are 2 ways to get the third outcome (H or T).
- The number of possible outcomes when a coin is tossed three times

$$= 2 \times 2 \times 2 = 8$$



Example 1:

The Matric Dance Committee has decided on the menu below for the 2020 Matric Dance. A person attending the dance must choose ONE item from each category; that is starters, main course and dessert.

Menu		
Starters	Main Course	Dessert
Crumbed Mushrooms Garlic Bread Fish	Fried Chicken Beef Bolognaise Chicken Curry Vegetable Curry	Ice-cream Malva Pudding

1. How many different meal combinations can be chosen?

   Solution: Starters: 3 choices; Main meal: 4 choices; Dessert: 2 choices

   Meal combinations $= 3 \times 4 \times 2 = 24$

2. A particular person wishes to have chicken as his main course. How many different meal combinations does he have?

   Solution: Starters: 3 choices; Main meal: 1 choice; Dessert: 2 choices

   Number of meal combinations: $3 \times 1 \times 2 = 6$

CAN YOU?

1. You would like to buy your first car. These are your choices.
   - There are 2 body styles:
     Sedan or
     Hatchback

   - There are **5** colours available:
     Red; blue;white; black and pale green
   - There are **3** models:
     GL (standard model),
     SS (sports model with bigger engine)
     SL (luxury model with leather seats)

   How many choices do you have?

Answer: 1. 30

Example 2: Consider the word COVID. You are required to arrange 5 letter word arrangements using the letters of the word.

1. The letters may be repeated

5	5	5	5	5

Number: $5 \times 5 \times 5 \times 5 \times 5 = 5^5$

5 spaces to be filled! Letters can be repeated. Each position can be filled in 5 ways!

2. The letters may not be repeated.

	4 letters left			
5	4	3	2	1

$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

This is known as 5 factorial!

CAN YOU?

2. Someone makes 5 digit numbers with the digits 1, 3, 5, 7 and 9.
   (a) How many different numbers can you make if there are no repeated digits?
   (b) How many different numbers can you make if repeated digits are allowed?
3. A grade 12 learner has an Accounting, Physics and Mathematics textbook. How many different ways can they be arranged on the bookshelf?

Answers:

2. (a) $5^5 = 3125$ (b) $5! = 120$
3. $3! = 6$

Tip: Use **SHIFT +** x^{-1}

Example 3: Given the digits 3, 4, 5, 6, 7, 8 and 9.

Calculate how many unique 5-digit numbers can be formed using the digits above, if:

(a) The digits may be repeated

(b) The digits may not be repeated.

Solution:

(a)

7 ways	7 ways	7 ways	7 ways	7 ways
Position 1	2	3	4	5

7 digits but only 5 places!

(a) $\therefore \mathbf{7^5 = 16\ 807\ \textit{ways}}$

(b)

7 ways	6 ways	5 ways	4 ways	3 ways

$\therefore \mathbf{7 \times 6 \times 5 \times 4 \times 3 = 2520\ \textit{ways}}$

We can also write it as:

$$\frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 2520$$

Easier to use the second form on the calculator

CAN YOU?

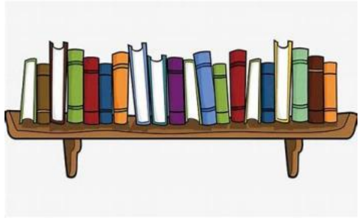
4. The new Gauteng number plate has two letters (from the alphabet A – Z, but not using vowels), two digits from 0 – 9 and then another two letters (excluding vowels).
   (a) How many possible combinations can be created of this number plate?
   (b) The previous Gauteng number plate had three letters (from the alphabet A – Z, but not using vowels) and then three digits from 0 – 9. Are there more combinations of the old or new number plates?

Answers:

(a) Number of possible combinations
$(21 \times 21) \times (10 \times 10) \times (21 \times 21) = 19\ 448\ 100.$

b) Number of possible combinations of old number plates:
$(21 \times 21 \times 21) \times (10 \times 10 \times 10) = 9\ 261\ 000.$
There are more combinations of the new number plate.

Now we are going to look at what happens if there is grouping:
Example 4:



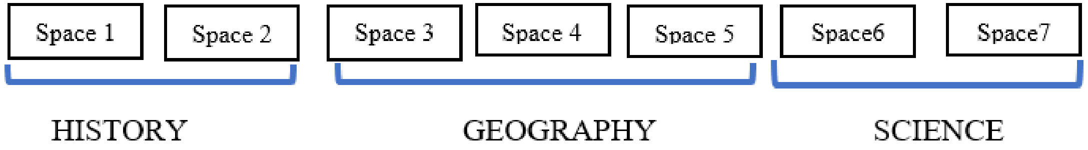
Two different history books, three different geography books and two different science books are placed on a bookshelf.

(a) How many ways can they be arranged?

(b) How many ways can they be arranged if books of the same subject must be placed together.

Solution:

(a) There are 7 books.
Therefore it can be arranged in 7! = 5040 ways



(b)
Let's look at the groups first. The 3 groups can be arranged in 3! ways
History books can be arranged in 2! ways
Geography can be arranged in 3! ways
Science can be arranged in 2! Ways
The total number of arrangements will be: $3! \times 2! \times 3! \times 2! = 144$

Can You?

5. Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

(a) In how many different ways can ALL the cars be parked?

(b) If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked.

Solution:

(a) $7! = 5\,040$

(a) The three silver cars can be parked in 3×2×1 ways.
[Forms 1 group]
Thus the group and the other 4 cars can be arranged in 5! ways.
So there are $(3!)(5!) =\ 720$ ways to park the cars.

Example 5.

A password of the form below, consists of six units. Where X is a letter in the alphabet and Y is a number (from 0 to 9).

X	X	Y	Y	X	X

(a) How many passwords can there be?

(b) How many different passwords can there be if X,Y and Z may not be used?

(c) How many different passwords are possible if you may use any letter or number only once?

(d) What is the probability of having a password that starts with AA?

(e) What is the probability of having a password that ends with an K and has only uneven numbers can only be used once?

Solution:

(a) $26 \times 26 \times 10 \times 10 \times 26 \times 26 = 45\ 697\ 600$

X: 26 letters in alphabet
Y: 10 options (0-9)

(b) $23 \times 23 \times 10 \times 10 \times 23 \times 23 = 27\ 984\ 100$

(c) $26 \times 25 \times 10 \times 9 \times 24 \times 23 = 32\ 292\ 000$

(d) Number of possibilities for AA: $1 \times 1 \times 10 \times 10 \times 26 \times 26 = 67600$

A: Only 1 option

$$P(AA\ at\ the\ beginning) = \frac{P(AA)}{P(total)} = \frac{67\ 600}{45\ 697\ 600} = \frac{1}{676}$$

(e) $26 \times 26 \times 5 \times 4 \times 26 \times 1 = 351\ 520$

Uneven numbers:5

$$P(Ending\ K\ and\ uneven\ no) = \frac{351\ 520}{45\ 687\ 600} = \frac{1}{130}$$

CAN YOU?

6. A password consists of six different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

(a) All the letters of the alphabet can be used.

(b) The password must start with a ‘D’ and end with an ‘L’

(c) Calculate the probability of a password starting with a ‘D’ and end with an ‘L’.

Solution:

(a) $26 \times 25 \times 24 \times 22 = 7\ 893\ 600$

(b) $1 \times 24 \times 23 \times 22 \times 1 = 12\ 144$

(c) Probability:

$$\frac{12\ 144}{7\ 893\ 600} = \frac{1}{650}$$

	Mind Action Series	Platinum	Clever	Everything Maths (Siyavula)	Classroom Maths
ACTIVITIES/ ASSESSMENT	Ex 1&2: Pg 310-312	Ex 4&5: Pg 124 Ex 11: Pg 135	Ex 13.2-13.4: Pg 361-367	Ex 10.4 : Pg 428 Ex 10.5: Pg 430	Ex 13.2-13.5; Pg359-365

CONSOLIDATION

- Read all question carefully
- Use place holders to assist you.
- Watch out for numbers not repeating and practice examples of grouping in an arrangement