SUBJECT and GRADE	Mathematics Grade 12	
TERM 1	Week 4	
TOPIC	Euclidean Geometry: Proportionality	
AIMS OF LESSON	• Prove the theorem that states that a line drawn parallel to one side of a triangle, divide the other two sides proportionally. • Answer riders using the proportionality theorem.	
RESOURCES	***Paper based resources***	***Digital resources***
	Mind the Gap; Your textbook	https://www.youtube.com/watch?cdlW3jwTj1Ag

INTRODUCTION

We need to revise the concept of ratio and proportion in order to be successful with this section of the work.

We know that a ratio compares 2 quantities of the same unit.

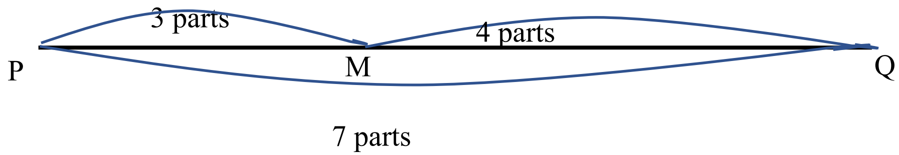
E.g. 1: 3 means 1 part to 3 parts

Mixing Oros: To make a perfect drink we need 1 part of Oros and 3 parts of water.

Remember that the unit must be the same.

Example 1:

If M divides the line PQ in the ratio 3:4 as shown below.



Note that the ratio is NOT the length of the line. We can use variables to indicate the equal parts of the ratio. Therefore $PM = 3k$ and $MQ = 4k$.

Determine the following ratios:

(a)	$\frac{\boldsymbol{PM}}{\boldsymbol{PQ}} = \frac{3k}{7k} = \frac{3}{7}$	(c)	$\frac{\boldsymbol{QM}}{\boldsymbol{MP}} = ?$
(b)	$\frac{\boldsymbol{PQ}}{\boldsymbol{QM}} = \frac{7k}{4k} = \frac{7}{4}$	(d)	$\frac{\boldsymbol{QP}}{\boldsymbol{PM}} = ?$
(e)	**The lengths of PM if PQ = 35 units?** $7k = 35\ units$ $k = 5\ units$ $PM = 3(5) = 15\ units$	(f)	**The lengths of MQ if PQ = 35 units?** $MQ = 4(5) = 20\ units$ $or\ \ MQ = 35 - 15 = 20\ units$

A **PROPORTION** is an equation of equivalent ratio's.

Using a small measuring unit in the ratio 1: 3 might not fill the glass to mix a glass of Oros. But we know that

$1{:}3\ =\ 4{:}12$ Remember that ratio's and proportion can be written in fractional form.

$\therefore \frac{1}{3} = \frac{4}{12}$

The special Fundamental Properties of a Proportion.

<table>
<tr><td rowspan="5">

If</td><td rowspan="5">$\frac{a}{b} = \frac{c}{d}$
$\frac{1}{3} = \frac{4}{12}$</td><td rowspan="5">then</td><td>$ad = bc$</td><td>$3 \times 4 = 1 \times 12$ Cross multiplication</td><td rowspan="5">

Handy tools!</td></tr>
<tr><td>$\frac{b}{a} = \frac{d}{c}$</td><td>$\frac{3}{1} = \frac{12}{4}$ Inverted</td></tr>
<tr><td>$\frac{a+b}{b} = \frac{c+d}{d}$</td><td>$\frac{1+3}{3} = \frac{4+12}{12} \therefore \frac{4}{3} = \frac{16}{12}$</td></tr>
<tr><td>$\frac{a}{c} = \frac{b}{d}$</td><td>$\frac{1}{4} = \frac{3}{12}$</td></tr>
<tr><td>$\frac{a-b}{b} = \frac{c-d}{d}$</td><td>` $-\frac{2}{3} = -\frac{8}{12}$</td></tr>
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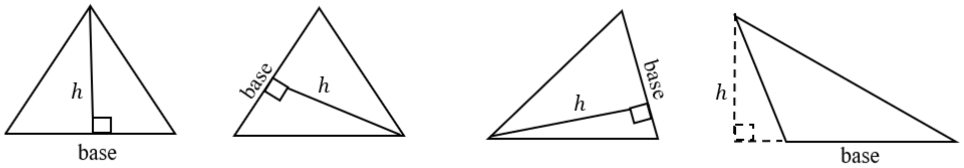
CONCEPTS AND SKILLS

Important information that we need before we can prove the first theorem.

Area of Triangles

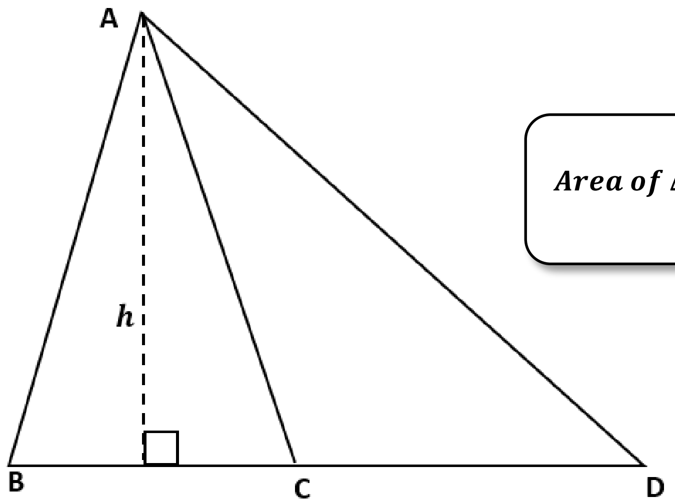
$$Area = \frac{1}{2}\ base\ \times \perp\ height$$



Note: The perpendicular height is dropped onto the base.

Therefore:

Triangles which share a common vertex have the same height.



$$\boldsymbol{Area\ of\ \Delta = \frac{1}{2}\ base \times \perp h}$$

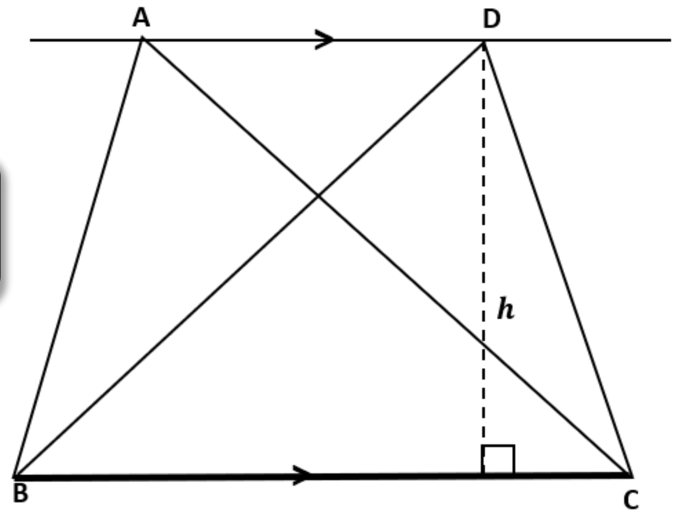
$$Area\ \Delta\ ABC = \frac{1}{2}\ BC \times h$$

$$Area\ \Delta\ ACD = \frac{1}{2}\ CD \times h$$

h the same.

Common vertex

Triangles on the same base and between the parallels are equal in area.



$$Area\ \Delta\ ABC = \frac{1}{2}\ BC \times h$$

$$Area\ \Delta\ DBC = \frac{1}{2}\ BC \times h$$

Area equal

Theorem 1.

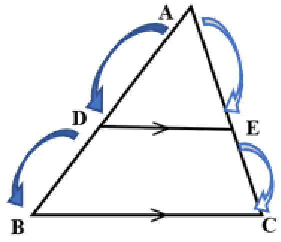
A line drawn parallel to one side of a triangle divides the other two sides proportionally.

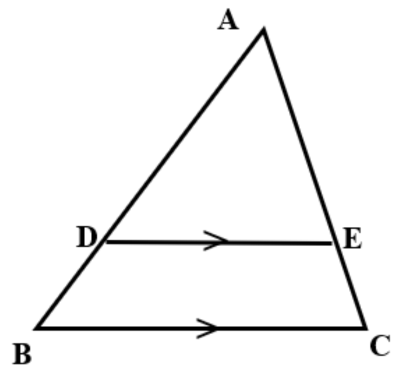
[prop theorem; DE ∥ BC]

Given: ΔABC with DE // BC.

Required to prove:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

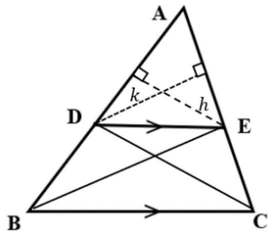




Construction:

Join BE and DC.

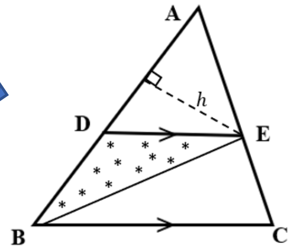
Draw height h relative to base AD and height k relative to base AE



Proof:

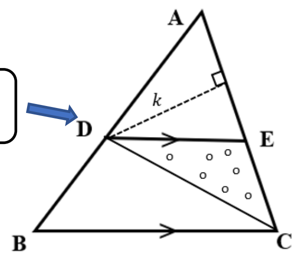
$$\frac{\text{Area } \Delta\text{ADE}}{\text{Area } \Delta\text{BDE}} = \frac{\frac{1}{2}AD \times h}{\frac{1}{2}DB \ \times h} = \frac{\text{AD}}{\text{DB}}$$

Common vertex E



$$\frac{\text{Area } \Delta\text{ADE}}{\text{Area } \Delta\text{CED}} = \frac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \ \times k} = \frac{\text{AE}}{\text{EC}}$$

Common vertex D

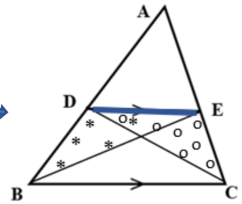


Area of ADE is common

Area Δ BDE = Δ CDE [same base DE and between //]

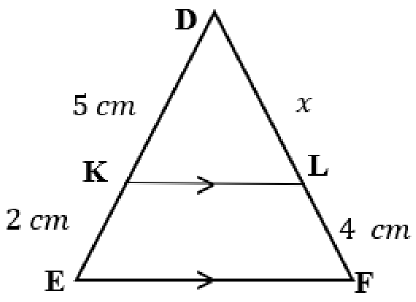
$$\frac{Area\ \Delta ADE}{Area\ \Delta BDE} = \frac{Area\ \Delta ADE}{Area\ \Delta CDE}$$

$$\therefore \quad \boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$$



Example 1

In Δ DEF, KL // EF. Calculate the value of x.



Solution.

$$\frac{DK}{KE}=\frac{DL}{LF}$$ [prop theorem; KL ∥ EF]

$$\frac{5}{2}=\frac{x}{4}$$

$$2x=20$$

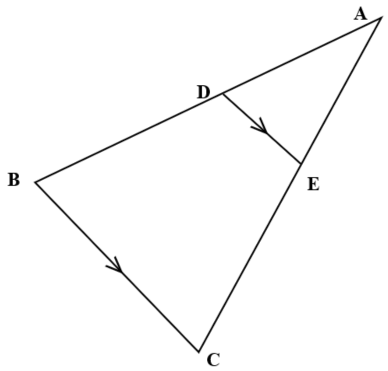
$$x=10\ cm$$

Example 2

In Δ ABC, DE ∥ AB, AB = 42 mm and

AE : EC = 3 : 4.

Determine the length of BD.



Solution:

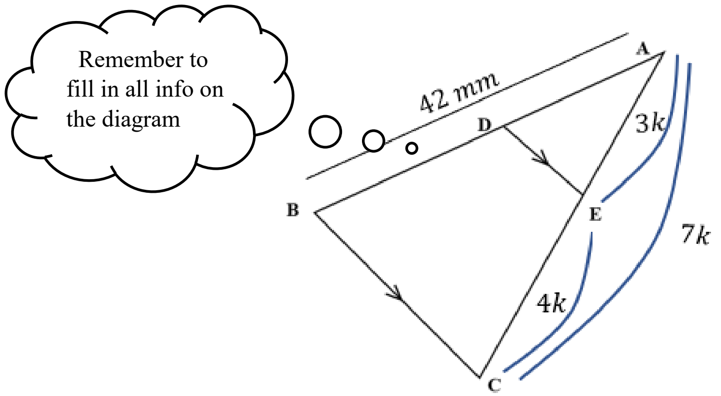
$$\frac{\text{BD}}{\text{AB}}=\frac{\text{CE}}{\text{AC}}$$ [prop theorem DE ∥ BC]

$$\frac{\text{BD}}{42}=\frac{4\text{k}}{7\text{k}}$$

$$\frac{\text{BD}}{42}=\frac{4}{7}$$

$$7\ \text{BD}=168$$

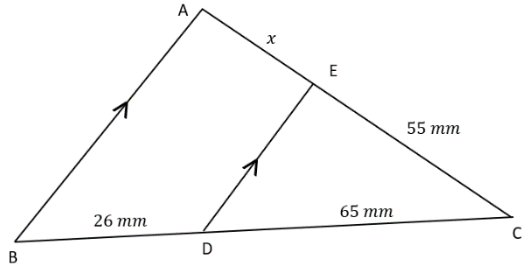
$$\text{BD}=24\ \text{mm}$$



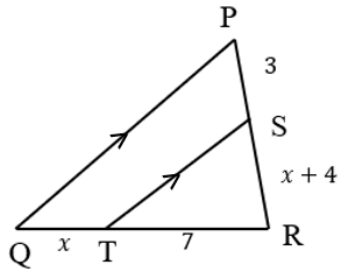
CAN YOU?

Calculate the length of sides labelled x and y in the following questions.

1.



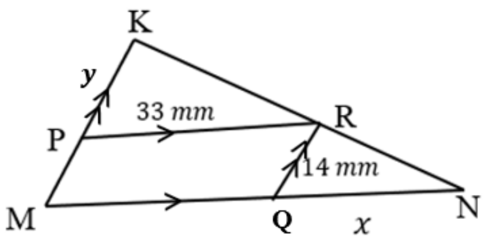
2.



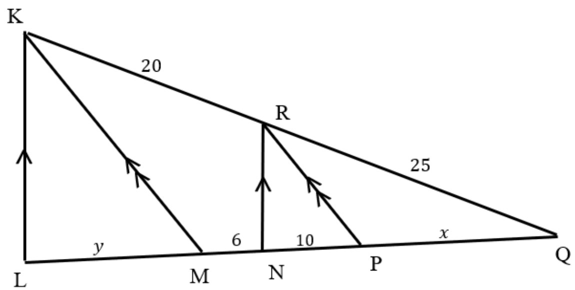
3. PMQR is a parallelogram

   PR=33mm, QR=14mm

   and KR : RN = 3 : 2



4.

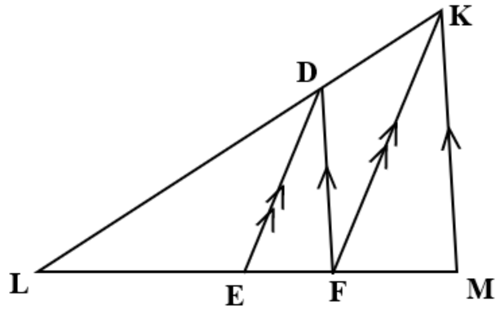


Example 3

In Δ KLM, KM ∥ DF, KF ∥ DE and

FE : EL = 3 : 4.

Determine LE : FM.



Solution:

$$\frac{KD}{DL} = \frac{MF}{LF} = \frac{MF}{7k} \quad \text{[prop theorem; DF ∥ KM]}$$

$$\therefore \frac{FE}{EL} = \frac{MF}{LF} \qquad \left[\text{both} = \frac{KD}{DL}\right]$$

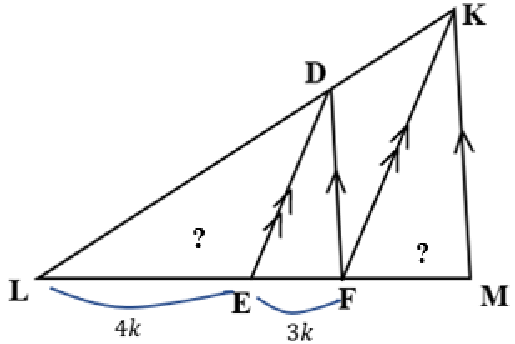
$$\frac{3k}{4k} = \frac{MF}{7k}$$

$$MF = \frac{21k^2}{4k} = \frac{21k}{4}$$

$$\therefore \frac{LE}{FM} = 4k \div \frac{21k}{4} = \frac{4k}{1} \times \frac{4}{21k}$$

$$= \frac{16k}{21k} = \frac{16}{21}$$

$\therefore$ FE : FM = 16 : 21

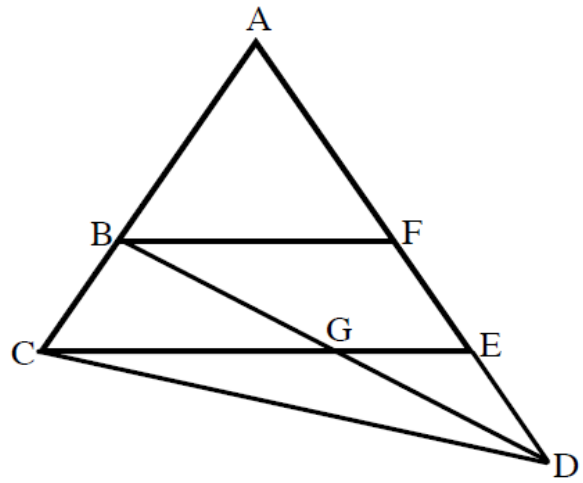


CAN YOU?

5.

In ΔACE, BF ∥ CE, $\frac{BC}{AC} = \frac{3}{8}$ and AE : ED = 4 : 3.

Determine DG: GB.



Solutions:

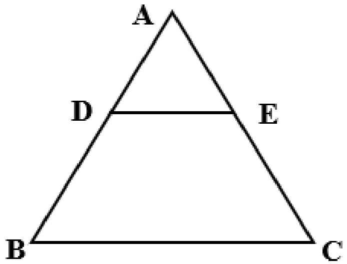
1. $x = 22$
2. $x = 3; x \neq -7$
3. $x = 22mm; y = 21mm$
4. $x = 20; y = 18$
5. 2 : 1

Converse Theorem

If a line cuts two sides of a triangle proportionally, then that line is parallel to the third side.

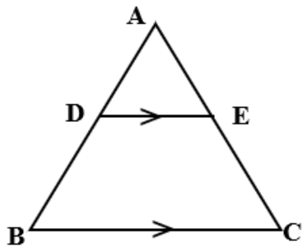
[line divides two sides of Δ prop]

If



$$\frac{AD}{DB} = \frac{AE}{EC}$$

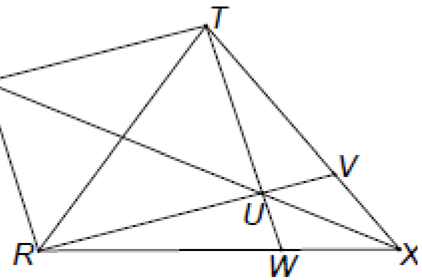
then



$$DE \parallel BC$$

CAN YOU?

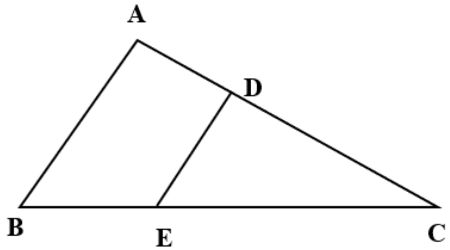
1. STUR is a parallelogram, with SUX, TUW and RUV straight lines. Prove RT ∥ VW.



Example 4

In ΔABC, AC = 13 cm, AD = 3 cm; BE = 3,6 cm and EC = 12 cm.

Prove that DE ∥ AB.



Solution:

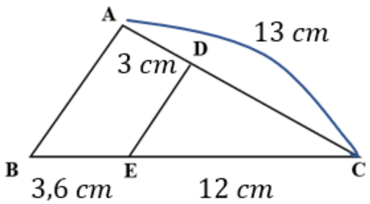
$$\frac{\text{BE}}{\text{EC}} = \frac{3{,}6}{12} = \frac{3}{10}$$

$$\text{DC} = 13 - 3 = 10 \text{ cm}$$

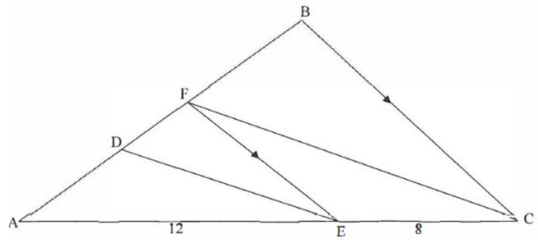
$$\therefore \frac{AD}{DC} = \frac{3}{10}$$

$$\therefore \frac{BE}{EC} = \frac{AD}{DC}$$

$\therefore DE \parallel AB$ [line divides two sides of Δ prop]



2. In the diagram ABC is a triangle with F on AB and E on AC. BC ∥ FE. D is on AF with, $\frac{AD}{AF} = \frac{3}{5}$, AE=12 units and EC = 8 units.

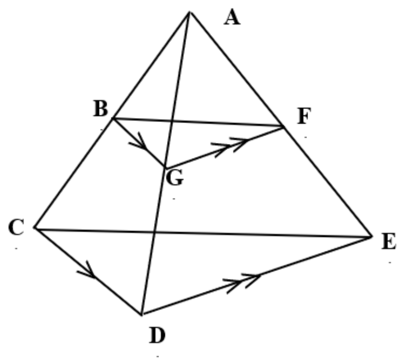


(a) Prove that DE ∥ FC.

(b) If AB = 14 units, calculate the length of BF. [**BF=5.6**]

Example 5

In Δ ACD and ΔADE, BG ∥ CD and GF ∥ DE. Prove that BF ∥ CE.



Solution:

$\frac{AB}{BC} = \frac{AG}{GD}$ [prop theorem; BG ∥ CD]

$\frac{AF}{FE} = \frac{AG}{GD}$ [prop theorem; GF ∥ DE]

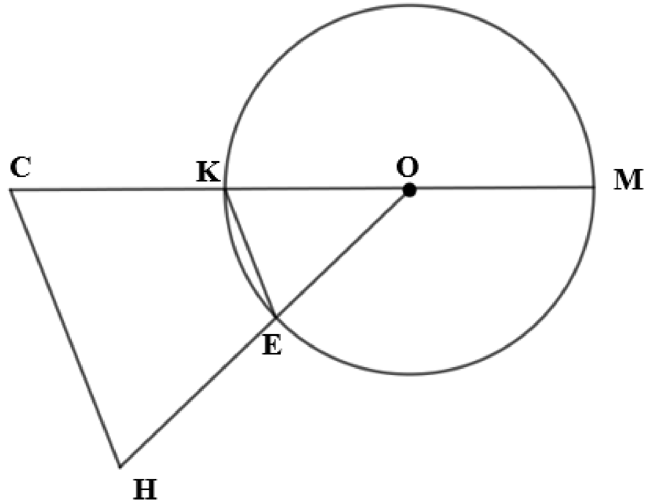
$\frac{AB}{BC} = \frac{AF}{FE}$ $\left[\text{both} = \frac{AG}{GD}\right]$

$BF \parallel CE$ [line divides two sides of Δ prop]

3. In the diagram below, KM is a diameter of a circle centre O.

OK= r. OC =4r and $\widehat{H} = \hat{C}$.

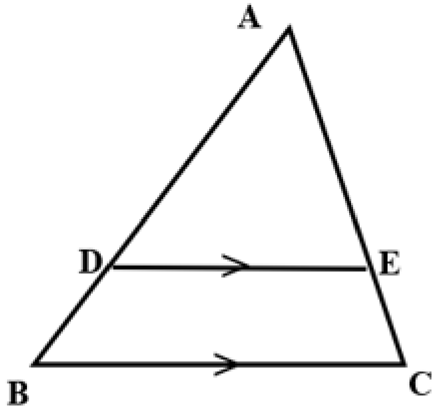
Prove that EK ∥ HC.

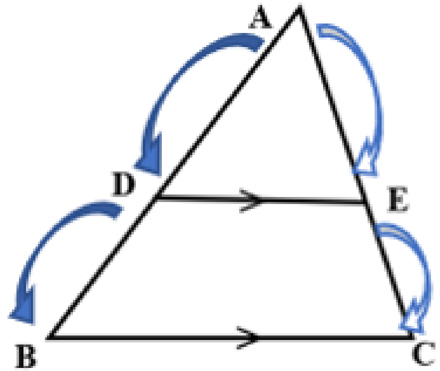


ACTIVITIES/ ASSESSMENT	Clever	Mind Action Series	Everything Maths Siyavula	Classroom Mathematics	Platinum
	Ex: 11.1; 11.2 Pg. 277	Ex: 1-3 Pg. 251	Ex: 8.4 -8.5 Pg. 329	Ex: 11.2 Pg. 287	Ex:2 Pg. 214

CONSOLIDATION

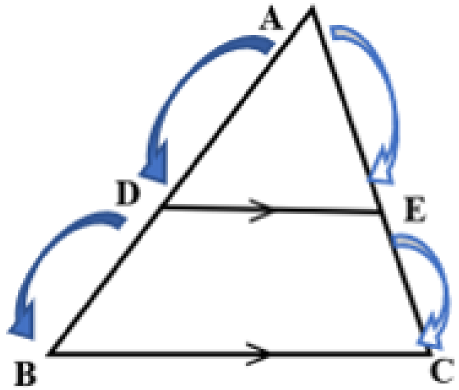
If **then**

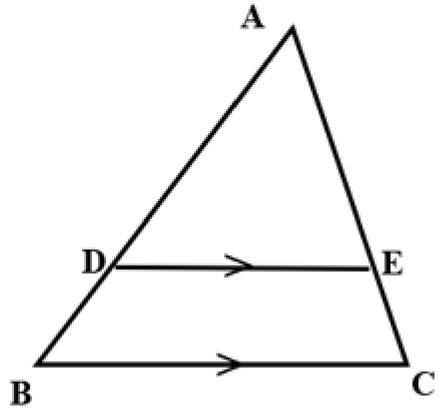




If $DE \parallel BC$ then $\frac{AB}{DB} = \frac{AE}{EC}$, $\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{BD}{AB} = \frac{EC}{AC}$ (as well as their inverses)

Converse





If $\frac{AB}{DB} = \frac{AE}{EC}$ or $\frac{AD}{AB} = \frac{AE}{AC}$ or $\frac{BD}{AB} = \frac{EC}{AC}$ or (any of these inverses) then $DE \parallel BC$