VAK EN GRAAD	Wiskunde Graad 12	
KWARTAAL 1	Week 4	
ONDERWERP	Euklidiese Meetkunde: Eweredigheid	
DOEL VAN LES	• Bewys die stelling, Die lyn ewewydig aan een sy van 'n driehoek verdeel die ander twee sye in eweredige dele. • Toepassing van die eweredigheid stelling.	
BRONNE	***Papier bronne***	***Digitale bronne***
	Mind the Gap; Jou handboek	https://www.youtube.com/watch?v=dlW3jeTj1Ag

INLEIDING

Ons moet die konsep van verhoudings en eweredighede hersien om suksesvol in hierdie gedeelte te wees.

Ons weet dat 'n verhouding twee hoeveelhede van dieselfde eenheid vergelyk.

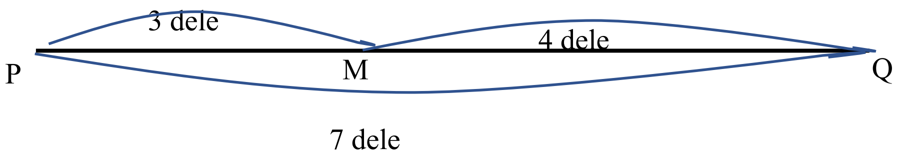
Eg. $1:3$ beteken 1 deel tot 3 dele

Die meng van Oros: Om 'n perfekte drankie te maak, benodig ons 1 deel Oros en 3 dele water.

Ons moet onthou dat die eenheid dieselfde moet wees.

Voorbeeld 1:

M verdeel die lyn PQ in die verhouding 3 : 4 soos hieronder aangetoon.



Let daarop dat die verhouding NIE die lengte van die lyn is nie. Ons kan veranderlikes gebruik om die gelyke dele van die verhouding aan te dui.
Dus is PM = 3k en MQ = 4k.

Bepaal die volgende verhoudings:

(a)	$\frac{\boldsymbol{PM}}{\boldsymbol{PQ}}=\frac{3k}{7k}=\frac{3}{7}$	(c)	$\frac{\boldsymbol{QM}}{\boldsymbol{MP}}=?$
(b)	$\frac{\boldsymbol{PQ}}{\boldsymbol{QM}}=\frac{7k}{4k}=\frac{7}{4}$	(d)	$\frac{\boldsymbol{QP}}{\boldsymbol{PM}}=?$
(e)	**Die lengte van PM as, PQ = 35, eenhede?** $7k=35\ eenhede$ $k=5\ eenhede$ $PM=3(5)=15\ eenhede$	(f)	**Die lengte van MQ as, PQ = 35 eenhede?** $MQ=4(5)=20\ eenhede$ $of\ \ MQ=35-15=20\ eenhede$

’n VERHOUDING is 'n vergelyking van ekwivalente verhoudings.

As u ’n klein maat instrument in die verhouding 1: 3 gebruik, sal dit nie genoeg wees om die glas met Oros te vul nie. Maar ons weet dat

1 : 3 = 4 : 12 Onthou dat verhoudings en eweredighede in breuk vorm geskryf kan word.

$\therefore \frac{1}{3}=\frac{4}{12}$

Die Spesiale Eienskappe van Eweredighede.

<table>
<tr><td rowspan="5">

As</td><td rowspan="5">$\frac{a}{b}=\frac{c}{d}$
$\frac{1}{3}=\frac{4}{12}$</td><td rowspan="5">dan is</td><td>$ad=bc$</td><td>$3\times 4=1\times 12$ kruis vermenigvuldiging</td><td rowspan="5">

Handige feite!</td></tr>
<tr><td>$\frac{b}{a}=\frac{d}{c}$</td><td>$\frac{3}{1}=\frac{12}{4}$ Omgekeer</td></tr>
<tr><td>$\frac{a+b}{b}=\frac{c+d}{d}$</td><td>$\frac{1+3}{3}=\frac{4+12}{12}\therefore\frac{4}{3}=\frac{16}{12}$</td></tr>
<tr><td>$\frac{a}{c}=\frac{b}{d}$</td><td>$\frac{1}{4}=\frac{3}{12}$</td></tr>
<tr><td>$\frac{a-b}{b}=\frac{c-d}{d}$</td><td>$-\frac{2}{3}=-\frac{8}{12}$</td></tr>
</table>

KONSEPTE EN VAARDIGHEDE

Belangrike inligting wat ons benodig voordat ons die eerste stelling kan bewys.

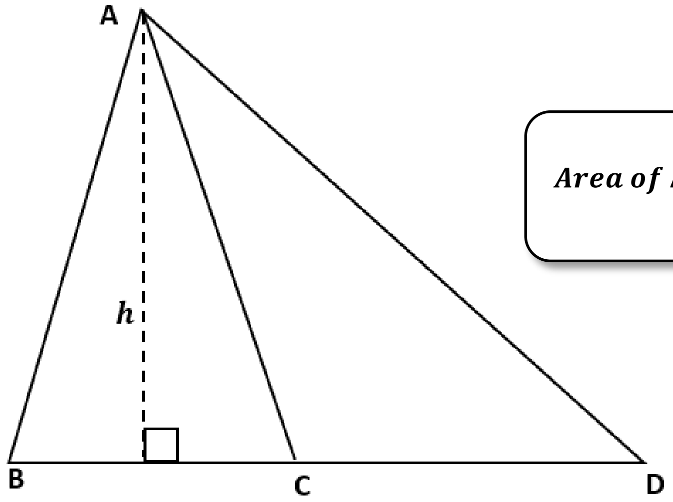
Area van Driehoek

$$Area = \frac{1}{2}\ basis\ \times \perp hoogte$$

Let op: Die loodregte hoogte is loodreg op die basis.

Dus:

Driehoeke met ’n gemene hoekpunt het dieselfde hoogtes.



$$\boldsymbol{Area\ of\ \Delta = \frac{1}{2}\ basis \times \perp h}$$

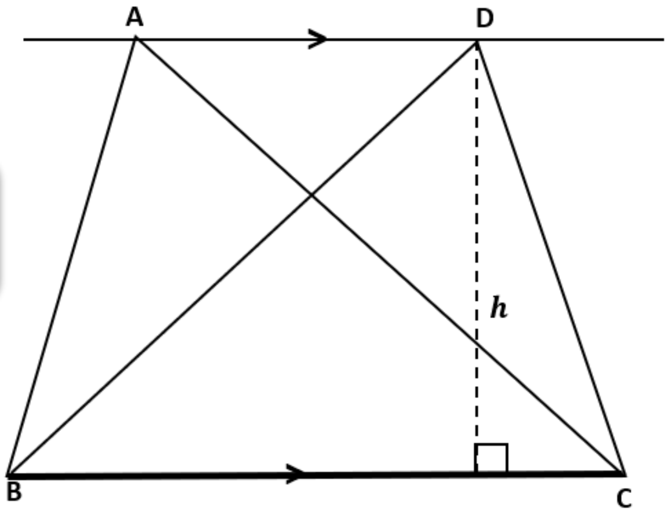
$Area\ \Delta\ ABC = \frac{1}{2}\ BC \times h$

$Area\ \Delta\ ACD = \frac{1}{2}\ CD \times h$

h is dieselfde.

Gemene hoekpunt

Driehoeke met ’n gemene basis en wat tussen dieselfde ewewydige lyne geleë is, is gelyk.



$Area\ \Delta\ ABC = \frac{1}{2}\ BC \times h$

$Area\ \Delta\ DBC = \frac{1}{2}\ BC \times h$

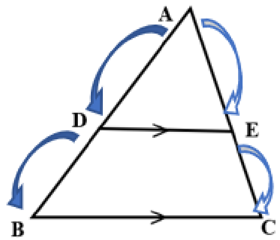
Area gelyk

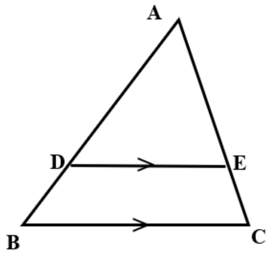
Stelling 1.

'n Lyn parallel aan een sy van 'n driehoek verdeel die ander twee sye eweredig.

[ewer stelling; DE ∥ BC]

Gegee: ΔABC met DE // BC.





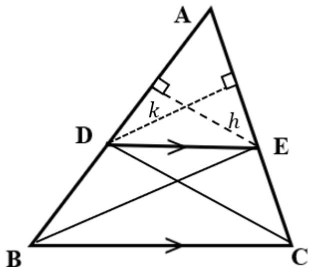
Te bewys:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Konstruksie:

Verbind BE en DC.

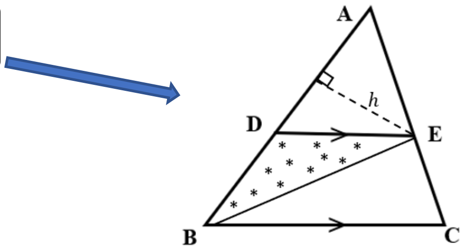
Teken hoogte h relatief tot basis AD en hoogte k relatief tot basis AE.



Bewys:

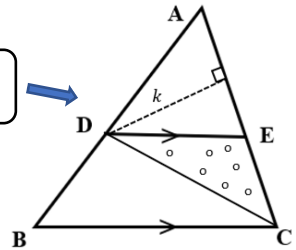
$$\frac{\text{Area } \Delta\text{ADE}}{\text{Area } \Delta\text{BDE}} = \frac{\frac{1}{2}AD \times h}{\frac{1}{2}DB \ \times h} = \frac{\text{AD}}{\text{DB}}$$

Gemene hoekpunt E



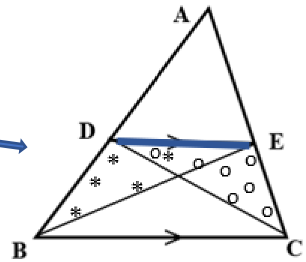
$$\frac{\text{Area } \Delta\text{ADE}}{\text{Area } \Delta\text{CED}} = \frac{\frac{1}{2}AD \times k}{\frac{1}{2}EC \ \times k} = \frac{\text{AE}}{\text{EC}}$$

Gemene hoekpunt D



Area van ΔADE is gemeen

Area Δ BDE = Δ CDE   [dieselfde basis DE en tussen //]



$$\frac{Area\ \Delta ADE}{Area\ \Delta BDE} = \frac{Area\ \Delta ADE}{Area\ \Delta CED}$$

$$\therefore \quad \boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$$

Voorbeeld 1

In Δ DEF, is KL // EF. Bereken die waarde van x.

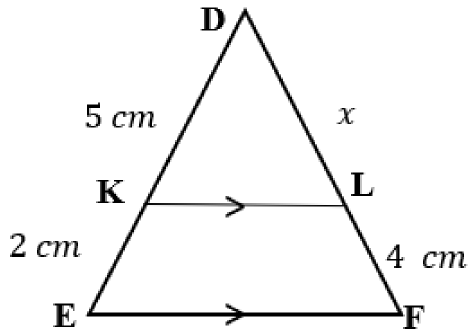
Oplossing.

$$\frac{DK}{KE} = \frac{DL}{LF} \qquad \text{[ewer stelling; KL ∥ EF]}$$

$$\frac{5}{2} = \frac{x}{4}$$

$$2x = 20$$

$$x = 10\ cm$$

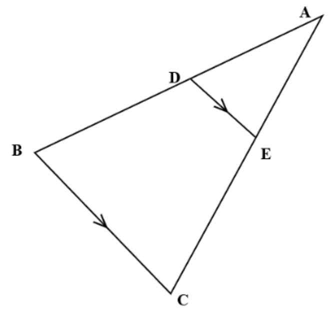


Voorbeeld 2:

In Δ ABC, is DE ∥ AB, AB = 42 mm en

AE : EC = 3 : 4.

Bepaal die lengte van BD.



Oplossing:

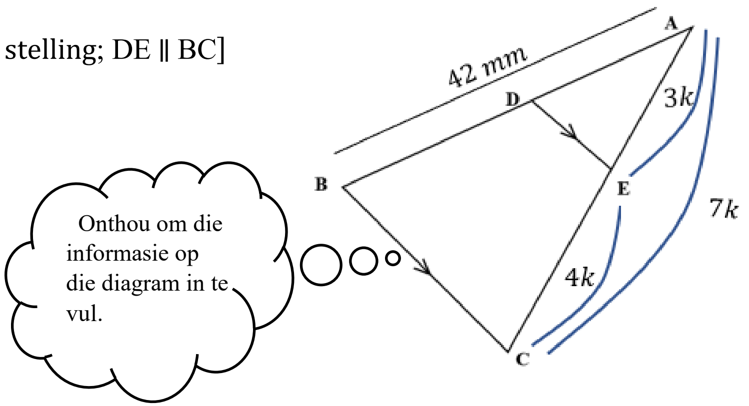
$$\frac{\text{BD}}{\text{AB}} = \frac{\text{CE}}{\text{AC}} \qquad \text{[ewer stelling; DE ∥ BC]}$$

$$\frac{\text{BD}}{42} = \frac{4\text{k}}{7\text{k}}$$

$$\frac{\text{BD}}{42} = \frac{4}{7}$$

$$7\ \text{BD} = 168$$

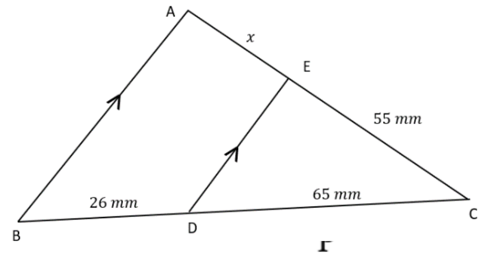
$$\text{BD} = 24\ \text{mm}$$



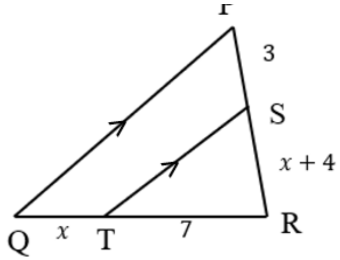
KAN JY?

Bereken die waarde van die sye, x en y in die volgende vrae.

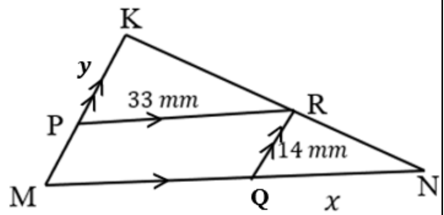
1.



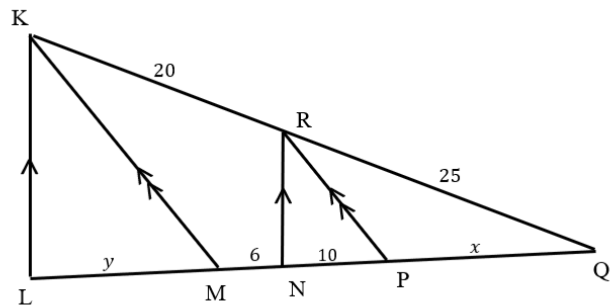
2.



3. PMQR is 'n parallelogram met PR=33mm, RQ = 14mm en KR : RN = 3 : 2



4.

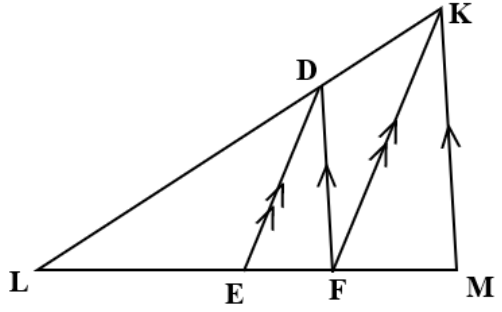


Voorbeeld 3

In Δ KLM, is KM ∥ DF, KF ∥ DE en

FE :EL = 3 : 4.

Bepaal LE : FM.



Oplossing:

$\frac{KD}{DL} = \frac{MF}{LF} = \frac{MF}{7k}$ [ewer stelling; DF ∥ KM]

$\therefore \frac{FE}{EL} = \frac{MF}{LF}$ $\left[\text{beide} = \frac{KD}{DL}\right]$

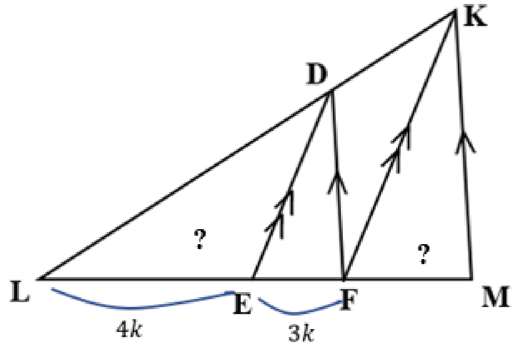
$\frac{3k}{4k} = \frac{MF}{7k}$

$MF = \frac{21k^2}{4k} = \frac{21k}{4}$

$\therefore \frac{LE}{FM} = 4k \div \frac{21k}{4} = \frac{4k}{1} \times \frac{4}{21k}$

$= \frac{16k}{21k} = \frac{16}{21}$

$\therefore$ FE : FM = 16 : 21

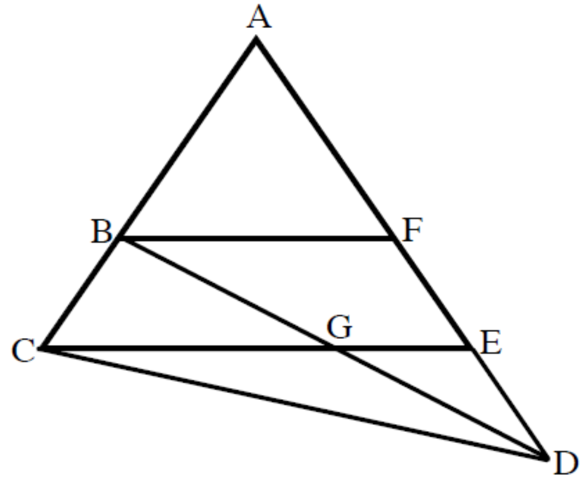


KAN JY?

5.

In ΔACE, is BF ∥ CE, $\frac{BC}{AC} = \frac{3}{8}$ en AE : ED = 4: 3.

Bepaal DG: GB.



Oplossings:

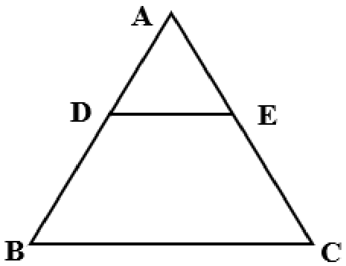
1. $x = 22$
2. $x = 3; x \neq -7$
3. $x = 22; y = 21$
4. $x = 20; y = 18$
5. 2:1

Omgekeerde Stelling

As 'n lyn twee sye van 'n driehoek eweredig verdeel, is die lyn parallel aan die derde sye van die driehoek.

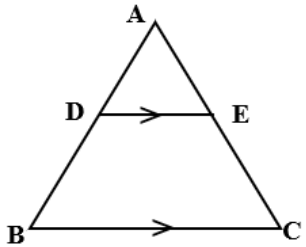
[lyn verdeel twee sye van Δ ewer]

As



$$\frac{AD}{DB} = \frac{AE}{EC}$$

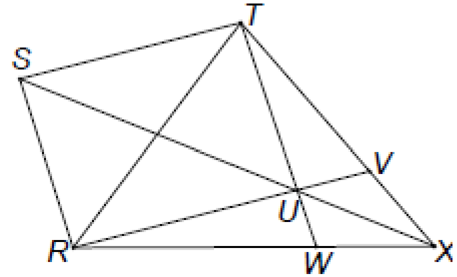
dan is



$$DE \parallel BC$$

KAN JY?

1. STUR is a parallelogram, met SUX, TUW en RUV reguit lyne. Bewys: RT ∥ VW.



Voorbeeld 4

In ΔABC, is AC = 13 cm, AD = 3 cm; BE = 3,6 cm en EC = 12 cm.

Bewys dat DE ∥ AB.

Oplossing:

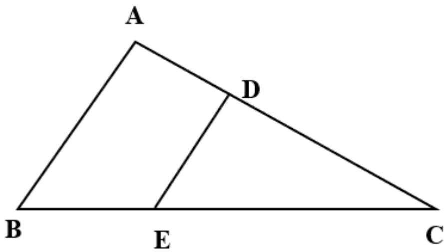
$$\frac{\text{BE}}{\text{EC}} = \frac{3{,}6}{12} = \frac{3}{10}$$

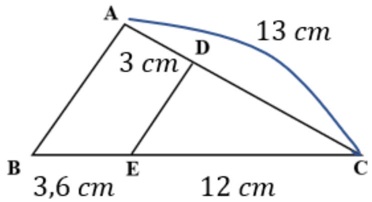
$$\text{DC} = 13 - 3 = 10 \text{ cm}$$

$$\therefore \frac{AD}{DC} = \frac{3}{10}$$

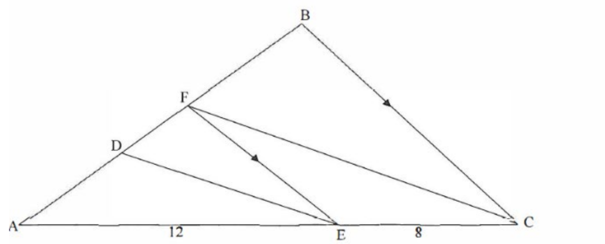
$$\therefore \frac{BE}{EC} = \frac{AD}{DC}$$

$\therefore DE \parallel AB$ [lyn verdeel twee sye van Δ ewer]





2. In die diagram ABC is 'n driehoek met F op AB en E op AC. BC∥ FE. D is op AF met, $\frac{AD}{AF} = \frac{3}{5}$, AE=12 eenhede en EC = 8 eenhede.



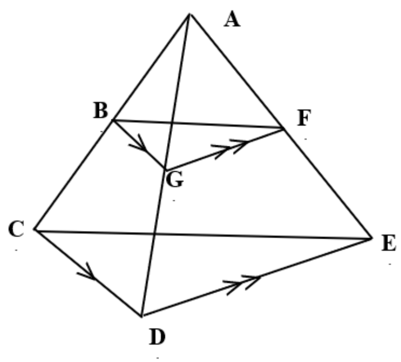
(a) Bewys dat DE ∥ FC.

(b) As AB = 14 eenhede is, bereken die lengte van BF.

[**BF = 5,6**]

Voorbeeld 5

In Δ ACD en ΔADE, is BG ∥ CD en GF ∥ DE. Bewys dat BF ∥ CE.



Oplossing:

$\frac{AB}{BC} = \frac{AG}{GD}$ [ewer stelling; BG ∥ CD]

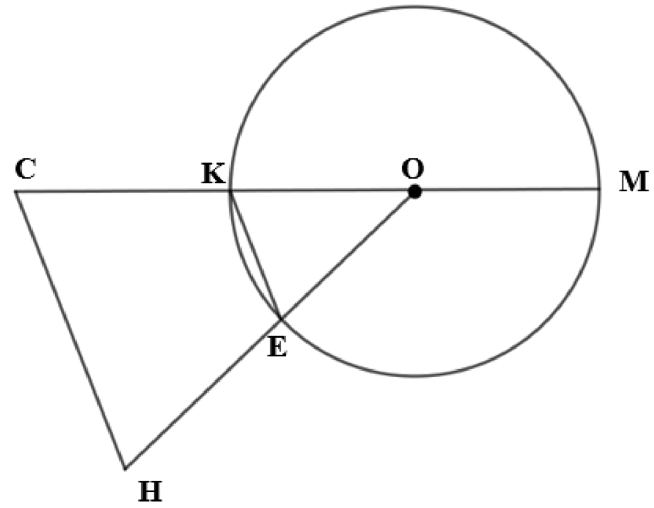
$\frac{AF}{FE} = \frac{AG}{GD}$ [ewer stelling; GF ∥ DE]

$\frac{AB}{BC} = \frac{AF}{FE}$ $\left[\text{beide} = \frac{AG}{GD}\right]$

$BF \parallel CE$ [lyn verdeel twee sye van Δ ewer]

3. In die diagram hieronder, is KM die middellyn van 'n sirkel met middelpunt O. OK= r. OC =4r en $\widehat{H} = \hat{C}$.

   Bewys dat EK ∥ HC.

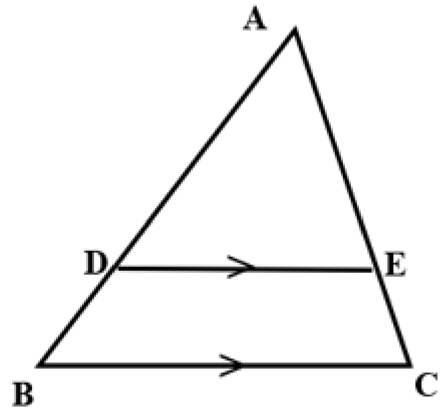


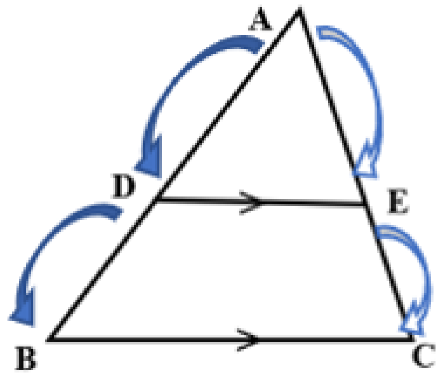
AKTIWITEITE/ ASSESERING

Clever	Mind Action Series	Everything Maths Siyavula	Classroom Mathematics	Platinum
Oef: 11.1; 11.2	Oef: 1-3	Oef: 8.4 -8.5	Oef: 11.2	Oef:2

KONSOLIDASIE

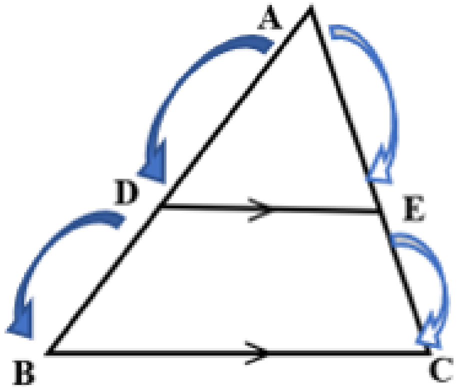
As **dan is**

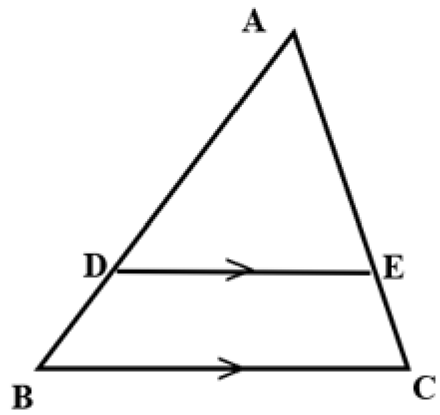




As $DE \parallel BC$ dan is $\frac{AB}{DB} = \frac{AE}{EC}$, $\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{BD}{AB} = \frac{EC}{AC}$ (as ook die omgekeerde)

Omgekeerde





As $\frac{AB}{DB} = \frac{AE}{EC}$ of $\frac{AD}{AB} = \frac{AE}{AC}$ of $\frac{BD}{AB} = \frac{EC}{AC}$ of (enige van hierdie se omgekeerde) dan is $DE \parallel BC$